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ILP heuristics and a new exact method for bi-objective 0/1 ILPs: Application to FTTx-network design

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ABSTRACT

Heuristics and metaheuristics are inevitable ingredients of most of the general purpose ILP solvers today, because of their contribution to the significant boost of the performance of exact methods. In the field of bi/multi-objective optimization, to the best of our knowledge, it is still not very common to integrate ILP heuristics into exact solution frameworks. This paper aims to bring a stronger attention of both the exact and metaheuristic communities to still unexplored possibilities for performance improvements of exact and heuristic multi-objective optimization algorithms.

We focus on bi-objective optimization problems whose feasible solutions can be described as 0/1 integer linear programs and propose two ILP heuristics, *boundary induced neighborhood search* (BINS) and *directional local branching*. Their main idea is to combine the features and explore the neighborhoods of solutions that are relatively close in the objective space. A *two-phase ILP*-based heuristic framework relying on BINS and directional local branching is introduced. Moreover, a new exact method called *adaptive search in objective space* (ASOS) is also proposed. ASOS combines features of the ϵ-constraint method with the binary search in the objective space and uses heuristic solutions produced by BINS for guidance. Our new methods are computationally evaluated on two problems of particular relevance for the design of FTTx-networks. Comparison with other known exact methods (relying on the exploration of the objective space) is conducted on a set of realistic benchmark instances representing telecommunication access networks from Germany.

1. Introduction

Exact multi-objective methods can be classified into *interactive* and *generating* ones, although the border between them is not very strict [1]. While interactive methods rely on an interaction between the decision maker and an algorithm, the generating methods aim at computing the complete Pareto front or a good representation set of it. Consequently, the latter methods are known to be computationally very demanding, given the potentially exponential size of Pareto fronts of underlying combinatorial optimization problems. Because of this, a great flourish of activities could be observed in the development of exact interactive methods, see for example [2,3] and a recent survey given in [1]. Among the exact methods, those derived using mathematical programming tools are of particular importance (see, e.g., [4], where an excellent overview of mathematical programming methods for multi-objective optimization can be found). As already predicted in this latter paper, rapid development of general purpose integer linear programming (ILP) solvers led to an increased popularity of exact ILP-based approaches for bi/multi-objective optimization.

Our paper focuses on generating mathematical programming methods, where, from the methodological perspective, two main research directions can be observed: branch-and-bound based algorithms and iterative exact methods, cf. [5]. The former perform the search in the decision space (see, e.g., [6–11]) while the latter perform the search in the objective/criterion space (see, e.g., [5,12,13]). A large body of work is available in the field of metaheuristics as well, in particular in the area of evolutionary multi-objective optimization (see, e.g., [14–16]). Not much has been done, however, in the development of *ILP-based heuristics* (or shortly, *ILP heuristics*) for bi-objective optimization. After many decades of independent research in mixed integer programming and metaheuristics for single-objective (combinatorial) optimization, researchers came upon realization that significant advantages

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can be drawn from synergetic effects of their hybridization. Nowadays, most of the general purpose ILP solvers contain (meta) heuristics as their inevitable features that also significantly contribute to the boost of their performance (see, e.g., [17–19]). In the field of bi/multi-objective optimization, this is still not the case, and the interaction between the communities is still fairly low. We found only very few examples in the recent literature, suggesting a hybridization of exact methods and metaheuristics (see, e.g., Section 4.4. of [15] and [20–22]). Hence, this paper is one of the first steps towards bringing a stronger attention of both communities to still unexplored possibilities for performance improvements of exact and heuristic multi-objective optimization methods.

In this paper we consider bi-objective combinatorial optimization problems that can be modeled as bi-objective 0/1 ILPs. Our contribution is twofold:

1. We propose two ILP heuristics for bi-objective 0/1 ILPs: *boundary induced neighborhood search* (BINS) and *directional local branching*, that are bi-objective counterparts of two efficient ILP heuristics for single-objective optimization, relaxation induced neighborhood search (RINS) [17] and local branching [18], respectively. The two ILP heuristics are then embedded into a *two-phase ILP-based heuristic* that is used to approximate the Pareto front for large instances.
2. We propose a new exact ILP-based method, *adaptive search in objective space* (ASOS) that explores the objective space in order to establish the complete Pareto front. This exact solution framework is based on combining the *binary search in objective space* (*BSOS*) [13,23] and the *ϵ-constraint method* [24]. Our framework is guided by (the absence of) heuristic solutions with the main goal to benefit from the advantages of the two methods while avoiding their individual drawbacks.

The development of these new methods is motivated by our computational experience with certain bi-objective problems arising in the design of FTTx-networks, showing that established iterative exact methods are not able to discover the complete Pareto front for most of the instances relevant for these practical applications.

Planning of telecommunication access networks: One main step in cost-efficient planning of telecommunication access networks is to find an (optimal) assignment of potential customers to different available *technologies* (*architectures*), i.e., a *deployment strategy*. Commonly used architectures include fiber-to-the-air (FTTA), fiber-to-the-curb (FTTC), fiber-to-the-building (FTTB), and fiber-to-the-home (FTTH). Network providers are faced with a natural question: which customers to serve with which technology so as to minimize the total investment costs while maximizing the quality of service. It is immediate that optimal deployment decisions are naturally subject to multiple objectives. Designing optimal FTTH networks is typically modeled as a variant of the *Steiner tree problem (STP)* in graphs (see, e.g., [25,26]) while variants of the *Connected Facility Location Problem (ConFL)* have been used for planning FTTC networks, cf. [27,28]. We introduce the *multi-objective k-architecture connected facility location problem* (MOkAConFL), generalizing connected facility location to more than two architectures and to multiple-objectives. The computational success of our new approaches is demonstrated on bi-objective problems, that arise as special cases of MOkAConFL with practical applications. These problems are the *bi-objective connected facility location problem* (BOConFL) and the *bi-objective two-architecture connected facility location problem* (BOTAConFL).

Outline of the paper: Required concepts from bi-objective optimization and necessary notation are summarized in Section 2. In this section, we also detail general concepts used in our implementations and give a short review of the BSOS and the ϵ-constraint method. Section 3 introduces our general-purpose ILP heuristics for the bi-objective case and discusses the new heuristic framework. Our new exact method, adaptive search in objective space, is detailed in Section 4. Section 5 introduces MOkAConFL, its bi-objective variants that will be used in our computational study, and details necessary for adaptating our frameworks to these particular problems. Further implementation details and the results of our computational study on the considered benchmark problems are summarized in Section 6. Finally, in Section 7, conclusions and possible directions for future research are provided.

2. Preliminaries

In this section, we introduce necessary notation and recall some basic terminology for bi-objective optimization, see, e.g., [29] for a more detailed overview. We also review two iterative exact methods and some generic speed-up techniques for iterative methods, which are used in both our heuristic and our exact framework.

2.1. Notation and terminology

Throughout this paper, we will only consider problems in minimization form and will assume that all input data is integral. For a bi-objective optimization problem $\min_{\sigma \in \mathcal{P}}(z_1(\sigma), z_2(\sigma))$, its feasible region $\mathcal{P}$ is called *decision space* and $Z = \{(z_1(\sigma), z_2(\sigma)) : \sigma \in \mathcal{P}\}$ is the set of images of the points in $\mathcal{P}$ in the *objective space* $\mathbb{R}^2$.

For ease of notation, for $\sigma^i \in \mathcal{P}$, let $z_1^i = z_1(\sigma^i), z_2^i = z_2(\sigma^i)$ and $z^i = (z_1^i, z_2^i)$. Moreover, we will also sometimes slightly abuse notation, and use z^i (i.e., a point in the objective space) to also refer to a solution σ^i (i.e., a point in the decision space) with $z_1(\sigma^i) = z_1^i$, $z_2(\sigma^i) = z_2^i$. This is only done when it is clear from the context, that such a solution exists.

A solution $\sigma^* \in \mathcal{P}$ is called *Pareto optimal (efficient)*, if and only if there is no solution $\sigma' \in \mathcal{P}$ such that $z_i(\sigma') \le z_i(\sigma^*)$, $i = 1, 2$, with at least one strict inequality. The objective point $z^* = (z_1(\sigma^*), z_2(\sigma^*))$ corresponding to an efficient solution σ^* is called *non-dominated*. The set of all Pareto optimal solutions is denoted by P_E and the set of all non-dominated points, also called *Pareto front* or non-dominated frontier, by $\mathcal{Z}$. An objective point $z(\overline{\sigma})$ corresponding to a solution $\overline{\sigma}$ is called *weakly dominated* iff there exists a Pareto optimal solution $\hat{\sigma}$ with $z_i(\hat{\sigma}) < z_i(\overline{\sigma})$ and $z_j(\hat{\sigma}) = z_j(\overline{\sigma})$, $i, j \in \{1, 2\}$, $i \ne j$, and there is no other Pareto optimal solution that strongly dominates $\overline{\sigma}$.

The set of efficient solutions can be partitioned into two subsets, those whose objective vectors lie on the boundary of the convex hull of the Pareto front, which are usually called *supported* efficient solutions, and the remaining, the so-called *non-supported* efficient solutions; the points in the objective space are called analogously. The boundary points (z_1^I, z_2^N) and (z_1^N, z_2^I) of the Pareto front that are defined by the *ideal point* $z_i^I = \min\{z_i(\sigma) : \sigma \in \mathcal{P}\}$ and the *nadir point* $z_i^N = \min\{z_i(\sigma) : \sigma \in \mathcal{P}, z_j(\sigma) \le z_j^I, j \ne i\}$, $i = 1, 2$, play an important role in most iterative solution methods. Given the objective vectors of two solutions σ^a and σ^b with $z_2(\sigma^a) > z_2(\sigma^b)$, we will denote by $[z^a, z^b]$ the *rectangle* $\{(z_1, z_2) | z_1^a \le z_1 \le z_1^b, z_2^b \le z_2 \le z_2^a\}$ in the objective space defined by these two solutions.

2.2. Iterative exact methods

As mentioned in the introduction, ILP-based generating exact methods for multi-objective optimization typically follow one of the two patterns: they either rely on the search of the decision space, or they establish the complete Pareto front by exploring the objective space. The methods studied in this paper fall into the latter category, and we will refer to them as *iterative methods*.