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Stochastic characterization of the spectrum sharing game in ad-hoc networks ☆

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ABSTRACT

This work focuses on infrastructure-less ad hoc wireless networks where multiple transmitter/receiver pairs share the same radio resources (spectrum); transmitters have to choose how to split a total power budget across orthogonal spectrum bands with the goal to maximize their sum rate under cumulative interference from concurrent transmissions. We start off by introducing and characterizing the non-cooperative game among transmitter/receiver pairs when the network topology is deterministically given. The corresponding Nash equilibria are derived, highlighting their dependency on the topological parameters (distances between wireless nodes, propagation model, and background noise power). The analysis is then extended to the case of random network topologies drawn from a given spatial stochastic process. Tools of stochastic geometry are leveraged to derive a statistical characterization of the equilibria of the spectrum sharing game. Finally, a distributed algorithm is proposed to let the players of the spectrum sharing game converge to equilibria conditions. Numerical simulations show that the proposed algorithm drives the users to stable points that are close to the equilibria of the game requiring limited information exchange among nodes.

1. Introduction

Resource sharing problems naturally arise in case of wireless networks where the transmission medium is inherently broadcast and transmission resources, e.g., frequency channels, transmission power levels, temporal slots, have to be orchestrated among multiple concurrent transmissions. The general goal is the design of sharing algorithms/protocol to maximize the "usage efficiency" of the shared resources.

In this work, we focus on infrastructure-less ad hoc wireless networks where multiple transmitter/receiver pairs share the same radio resources (spectrum). Each pair is allowed to allocate an available transmission power budget across multiple radio resources for transmission (reception). Such scenario well represents those practical cases where either multiple Network Interface Cards (NICs) are available at each transmitter/receiver [3] or where the transmission technology allows to use different power levels on different subcarriers, like the IEEE 802.11a OFDMA-based technology [4]. In any case, the "quality" perceived by a transmitter/receiver pair is influenced by the behavior (resource allocation) of the other users. The reference scenario is inherently distributed as there is no central entity to coordinate the resource allocation network-wide, but rather each transmitter/receiver pair

☆ This work extends our preliminary study presented in [1,2].

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adopts local resource allocation policies depending of its local view and knowledge of the network. In this view, our main goals are to tell whether (and when) there exist equilibrium policies which are stable; whether these equilibria are unique; and whether they can be reached by distributed algorithms. To this extent, we model the interaction among users as a non-cooperative game.

In particular, we consider a Gaussian Interference Game (GIG) [5], where multiple transmitters have to decide how to split a total power budget across orthogonal spectrum bands with equal bandwidth; the users play to maximize their sum rate over all the used bands which depends on the interference level produced by the other users and some background Gaussian noise. Since the wireless network performance is strongly influenced by the spatial distribution of the communicating/interfering nodes, a natural objective is to analyze the dependency of the equilibria distribution on the node positions. To this extent, we start off by introducing and characterizing the non-cooperative game where the network topology is deterministically given, that is, the relative positions of transmitters and receivers are known. We take here a constructive approach by analyzing at first a simple but insightful network topology with two transmitter/receiver pairs. In this scenario, we derive the quality/number of the Nash equilibria, highlighting their dependency on the topological parameters (distances between wireless nodes, propagation model, and background noise power). We show that the game solution features a bi-modal behavior in which if the two pairs are "far enough" the equilibrium is unique and coincide with the optimal centralized-based allocation, whereas if the two pairs "get closer" the game admits multiple equilibria that in general do not coincide with the optimum.

The paper then moves to the characterization of the non-cooperative spectrum sharing game for random network topologies drawn from a given spatial stochastic process. We leverage tools of stochastic geometry to derive a statistical characterization of the equilibria of the spectrum sharing game. Again, we start off by providing the analysis of the equilibria in the two transmitter/receiver pairs scenario; we further show how the analysis of the 2-player game can be leveraged to characterize the N-player game. Namely, the analysis of large networks game can be simplified, by wisely decomposing the N-player game into independent equivalent sub-games of two players. A heuristic algorithm is proposed to implement such decomposition, thus allowing to get the equilibria for the N-player game. Finally, a distributed algorithm is introduced to let the N players converge to equilibria conditions. Numerical simulations show that the proposed algorithm drives the users to stable points that are close to the equilibria of the game further requiring only minimal information exchange.

In short, the main contributions of the present work can be summarized as follows:

1. Analysis of the 2-player spectrum sharing game.
   (a) Characterization for deterministic topologies of the Nash equilibria, their stability and quality with respect to the optimal solution.
   (b) Analysis of the spectrum sharing game in stochastic topologies.
2. Analysis of the N-player spectrum sharing game.
   (a) Proof of existence of at least one Nash equilibrium in pure strategy.
   (b) Analysis through directed influence graph approach and coupling probability.
3. Distributed algorithm to drive the selfish user to play equilibria strategies in spectrum sharing.

The paper is organized as follows: in Section 2 we review prior work on game theoretic approaches to spectrum/resource sharing in wireless networks; Section 3 sets the reference scenario; Section 4 provides a detailed analysis of the 2-player game; Section 5 extends the analysis to the N-player game; Section 6 provides distributed algorithm to reach equilibrium and compare the result with the game solutions; and Section 7 concludes the paper.

2. Related work

The relevant literature includes work on game theoretic tools and stochastic geometry applied to network scenarios where network nodes have to optimally allocate transmitted power across multiple (semi-) orthogonal bands with target of maximizing the achievable throughput. Such problem is indeed relevant in different networking scenarios including ad hoc networks, cognitive radio networks and wireless access networks. In the following, we give a succinct overview of the main findings and approaches related to these three application scenarios. In [6], the authors consider a power control problem with SINR as objective function, in both selfish and cooperative scenario. In the selfish case, the users play to maximize their achievable average SINR over all the resources they use to transmit; existence and "quality" of the Nash equilibria are studied under different cases for mutual interference between the users over the available resources. In the cooperative scenario, the users play to maximize the sum average SINR.

Wang and Krunz consider in [7] a non cooperative scenario where users compete in the power allocation to maximize their performances. A pricing mechanism to steer the non-cooperative spectrum sharing process towards optimal equilibria is proposed.

Power games based on the water-filling algorithm are proposed in [8,9]. The authors consider a scenario composed by two contending communicating systems and study the existence and uniqueness of the Nash equilibrium. The water-filling algorithm is used also in [10,11]. A unified view of main results presented in the literature is proposed in [12]. This work shows how the different approaches proposed in the literature can be unified following a unique interpretation of the water-filling solution. Furthermore, a unified set of sufficient conditions that guarantee the uniqueness of the equilibrium is derived. Baidas and McKenzie [13] leverage auction mechanisms to properly set the transmission power levels in multi-source, multi-relay wireless networks in a cooperative environment.