Theoretical Computer Science 508 (2013) 16–25

Contents lists available at ScienceDirect

Theoretical Computer Science

journal homepage: www.elsevier.com/locate/tcs

Parameterized complexity of Min-power multicast problems in wireless ad hoc networks ☆

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A R T I C L E   I N F O

Keywords:
Power assignment
W[1]-/W[2]-hardness
Fixed-parameter tractability
Lower bound

A B S T R A C T

Power assignment in wireless ad hoc networks can be seen as a family of problems in which the task is to find in a given power requirement network a minimum power communication subnetwork that satisfies a given connectivity constraint. These problems have been extensively studied from the viewpoint of approximation, heuristic, linear programming, *etc*. In this paper, we add a new facet by initiating a systematic parameterized complexity study of three types of power assignment problems related to multicast: Min-power Single-source h-Multicast, Min-power Strongly Connected h-Multicast and Min-power Multi-source h-Multicast. We investigate their parameterized complexities with respect to the number of terminals and the number of senders. We show that a Min-power Single-source h-Multicast is fixed-parameter tractable with respect to the number of terminals and achieve several parameterized hardness results.

1. Introduction

Power assignment is a central issue in wireless ad hoc networks. The main goal of power assignment is to assign transmission power to the wireless nodes so as to guarantee communication between the nodes, while minimizing the overall power consumption of the network. Depending on the communication requirement such as broadcast, multicast, and strong connectivity, power assignment problems come in many flavors, most of them having been proved to be NP-hard. Thus, tremendous research on power assignment has been conducted from the viewpoint of approximation, heuristic, linear programming, etc. By way of contrast, the study of the parameterized complexity of these problems is much less developed. In this work, we investigate the parameterized complexity of three types of power assignment problems. We first introduce the considered problems.

Problem statements. The wireless ad hoc network has no pre-installed infrastructure and is formed by a collection of wireless nodes equipped with omnidirectional antennas for sending and receiving signals. Power is one of the most critical resources and energy efficiency is an important design consideration for these networks. To save energy, each node can adjust its power levels, based on the distance to the receivers and the background noise.

☆ A preliminary version of this paper appeared in Proc. of FAW-AAIM (2011) [17]. This work is supported by the National Natural Science Foundation of China under Grant (61073036, 61128006), the Doctoral Discipline Foundation of Higher Education Institution of China under Grant (20090162110056), and the DFG Excellence Cluster "Multimodal Computing and Interaction (MMCI)".

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0304-3975/$ – see front matter
doi:10.1016/j.tcs.2012.02.040

Formally, let $M = (N, A, c)$ be a power requirement network, where N represents the set of wireless nodes with $|N| = n$, A the set of communication links between the nodes and a *power requirement* function $c : A \to R^+$. Each directed link $[x_i, x_j] \in A$ has link cost $c([x_i, x_j])$ which is the minimum power that is necessary to sustain link $[x_i, x_j]$. Each node $x_i \in N$ can choose to transmit at different power levels. Let P_{x_i} be the set of power levels at which node x_i can transmit ($|P_{x_i}|$ can be arbitrarily large). Let $P = \bigcup_{x_i \in N} P_{x_i}$. Given a power assignment function $p : N \to P$, a directed link $[x_i, x_j]$ is supported by p if $p(x_i) \geq c([x_i, x_j])$. The communication subnetwork H of M consists of all supported links.

Power Assignment problems

Input: A power requirement network $M = (N, A, c)$ with $|N| = n$, a family of power level sets $P_{x_1}, P_{x_2}, \ldots, P_{x_n}$ and a connectivity constraint Q.

Task: Find a power assignment $p : N \to P$ ($P = \bigcup_{x_i \in N} P_{x_i}$) of the minimum total power cost $\sum_{x_i \in N} p(x_i)$ such that the communication subnetwork H satisfies the given connectivity constraint Q and $p(x_i) \in P_{x_i}$ for every node $x_i \in N$.

Multicast is an efficient mechanism for one to many communication. In this paper, we focus on the following connectivity constraints Q related to multicast for the communication subnetwork H:

(1) Bounded hop single source multicast (D, r, h) where $D \subseteq N$, $r \in N$, and h is a non-negative integer: There is a directed path in H from r to each node $v \in D$ of length at most h.

(2) Bounded hop strongly connected multicast (D, h) where $D \subseteq N$ and h is a non-negative integer: For every $s, t \in D$ there is a directed path in H from s to t of length at most h.

(3) Bounded hop multi-source multicast (D, S, h) where $D \subseteq N$, $S \subseteq N$, and h is a non-negative integer: For each $r \in S$, there is a directed path in H from r to each node $v \in D$ of length at most h.

We call the nodes in D *terminals*, a node v in H *sender* if the out-degree of v is at least one, and h the *hop* of H.

Specifying the connectivity constraint, we obtain the following problems: Min-power Single-source h-Multicast, Min-power Strongly-connected h-Multicast, and Min-power Multi-source h-Multicast.

Known Results. The above three multicast-related power assignment problems are NP-hard even under the constraint that the transmission power at each node is fixed or finitely adjustable in a discrete fashion [19,6,16]. The approximability of power assignment problems has received considerable attention. Liang [15] proposed an approximation algorithm with ratio $4 \ln k$ (k is the number of terminals) for a Min-power Single-source Multicast (hop is unlimited) where the transmission power at each node has limited adjustable discrete power levels. In [16], the authors proposed several approximation algorithms for a Min-power Multi-source Multicast problem where the transmission power is either fixed or adjustable. Calinescu et al. [6] gave an approximation algorithm with ratio $3 + 2\ln(n - 1)$ for the power assignment with the strong connectivity constraint. In particular, many papers have studied approximation algorithms for power assignment problems where hops or senders are bounded [1,7,13,14,20]. See [5] for a survey on numerous approximation results for power assignment problems.

Our works. We initialize a systematic study of the parameterized complexity of the above three power assignment problems. We assume that each node can choose to transmit messages from a limited number of discrete power levels, i.e., $|P_{x_i}|$ is limited for any node $x_i \in N$.

Two natural parameters are considered: (1) the parameter k_1 that denotes the number of terminals to be connected, (2) an upper bound k_2 on the number of the senders in the solution H. We prove that a Min-power Single-source h-Multicast is fixed-parameter tractable with respect to k_1, but is W[2]-hard with respect to k_2. Moreover, we show that a Min-power Single-source h-Multicast with respect to (k_1, k_2) does not admit a polynomial kernel. For a Min-power Strongly-connected h-Multicast, we obtain W[1]-hardness with respect to the combined parameter (k_1, k_2). Since a Min-power Multi-source h-Multicast is a generalization of a Min-power Single-source h-Multicast, a Min-power Multi-source h-Multicast is also W[2]-hard with respect to k_2. The parameterized complexity of a Min-power Multi-source h-Multicast with respect to k_1 remains open.

2. Graph model and related definitions

Graph model. For simplicity of exposition, based on the wireless network model, we give an equivalent graph model of the power assignment problems. Let $G = (V, E)$ be a directed graph with weighting function $weight$: $E \to R^+$ and let $e = [u, v]$ be a directed edge in G. We say that the edge e goes out from vertex u and comes into the vertex v. The edge e is called an outgoing edge of the vertex u, and an incoming edge of the vertex v. Moreover, u is an in-neighbor of v and v is an out-neighbor of u. For two vertices $v, w \in V$, let $d[v, w]$ be a directed simple path from v to w. The length of a simple path is the number of edges in the path. For a vertex $w \in V$, let $out\text{-}degree(w)$ be the number of outgoing edges of w, $out(w)$ be the set of outgoing edges of w and $out\text{-}weight(w) = \{weight(e) | e \in out(w)\}$. Similarly, for a vertex $w \in V$, let $in\text{-}degree(w)$ be the number of incoming edges of w, $in(w)$ be the set of incoming edges of w. For a vertex $v \in V$, let $N_{out}(v) = \{w | [v, w] \in E\}$. Given a subgraph H of G, let $NL(H) = \{w | w \in V(H)$ and $out\text{-}degree(w) \geq 1\}$, and the vertices in $NL(H)$ are called *senders*. For $w \in V$, let $weight_{max}(w) = \max\{weight(e) | e \in out(w)\}$. For a subgraph H of G, we define $cost_w(H) = \sum_{u \in NL(H)} weight_{max}(u)$ and $cost(H) = \sum_{e \in E(H)} weight(e)$.