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A game theory model of urban public traffic networks

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Abstract

We have studied urban public traffic networks from the viewpoint of complex networks and game theory. Firstly, we have empirically investigated an urban public traffic network in Beijing in 2003, and obtained its statistical properties. Then a simplified game theory model is proposed for simulating the evolution of the traffic network. The basic idea is that three network manipulators, passengers, an urban public traffic company, and a government traffic management agency, play games in a network evolution process. Each manipulator tries to build the traffic lines to magnify its "benefit". Simulation results show a good qualitative agreement with the empirical results.

Keywords: Network; Game theory; Urban public traffic

1. Introduction

Complex networks can describe variety different practical systems [1–5]. It may serve as a useful tool for understanding complex systems. One way to study complex networks may be performing empirical investigation first and then set up suitable models to understand the common dynamical characteristics and mechanism of the systems. Since urban public traffic systems are practically important, they have been extensively and intensively studied in recent years [6–11]. Among the studies, [6] suggests a network description for both the urban public traffic systems in Beijing and Yangzhou. Each bus line is defined as a "collaboration act" and each bus station as a node ("actor"). An edge is connected between a pair of bus stations if they take part in one bus line.

We suggest, in this paper, that the urban public traffic company, the passengers and the government traffic management agency take most important roles on the urban public traffic network evolution. Each of the three manipulators competes with others to magnify its "benefit". The urban public traffic company firstly considers its income benefit, the passengers may only consider their convenience and ticket expenses, and the government traffic management agency may consider the effectiveness and public safety. Meanwhile, they know that the game will be over if any of the three gives up, so they prepare to give in if any side cannot bear. This keeps the collaboration between them. The evolution process of the network may be described by such a

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process, in which the three manipulators play games, and finally it reaches an equilibrium, which leads to a situation where the network benefits all the three manipulators equally. Based on this idea, in this paper we suggest a simplified network evolution model. To our knowledge, this is the first discussion of a manipulator game in a network. This idea may be suitable for description of some other systems evolution.

Section 2 presents the statistical properties of urban public traffic network in Beijing in 2003. Section 3 presents our model and its evolution. The simulation results are shown in Section 4. A summary and discussion are given in the last section.

2. Empirical investigation

We have done an empirical investigation on the urban public traffic network in Beijing (UPTN-B) in 2003. It includes 65 urban bus lines and 460 bus stations. We define each bus station as a node; two nodes are connected by an edge if they appear in the same bus line.

Fig. 1 shows the distribution of accumulative act-size (the number of bus stations in a bus line) for UPTN-B. In the figure, the vertical axis represents the probability of accumulative act-size and the horizontal axis, a certain number of the act-size. The solid cycles denote the empirical data. The solid curve shows that the data can be fitted by a shifted Poisson distribution, which may be expressed as $p_\lambda(T) = [\lambda^T/(A\,T!)]e^{-\lambda}$, where λ (= 55.9 here) denotes the maximum of the function, T the integers equal or larger than λ, $1/A$ the normalizing factor [6].

Fig. 2 shows the distribution of accumulative act-degree (the number of bus lines, which passengers can take in a bus station) for UPTN-B. In the figure the vertical axis represents the probability of accumulative act-degree, and the horizontal axis, a certain number of the act-degree. The solid cycles denote the empirical data. The solid line the least-square fitting. Fig. 3 presents the distribution of accumulative degree (the number of bus stations to which passengers can reach by direct bus lines) of UPTN-B. In the figure the vertical axis represents the probability of accumulative degree and the horizontal axis, a certain number of the degree. The void cycles denote the empirical data and the solid line, the least-square fitting. Both of the distributions can be well described with exponential functions.

Furthermore, we have also obtained the distributions of the multiple edge numbers (the number of direct bus lines, which passengers can take in this station for all the adjacent stations) and the distribution of the node-pair distances (the number of the edges between the pair of stations) from the empirical investigation data. The distribution of the multiple edge numbers shows an exponential decay, and the distribution of the node-pair distances displays a shifted Poisson distribution.

In addition, the empirical investigations for several urban public traffic networks in 2006 have also been collected. These investigations include Beijing (572 bus lines and 4199 bus stations including suburb lines),

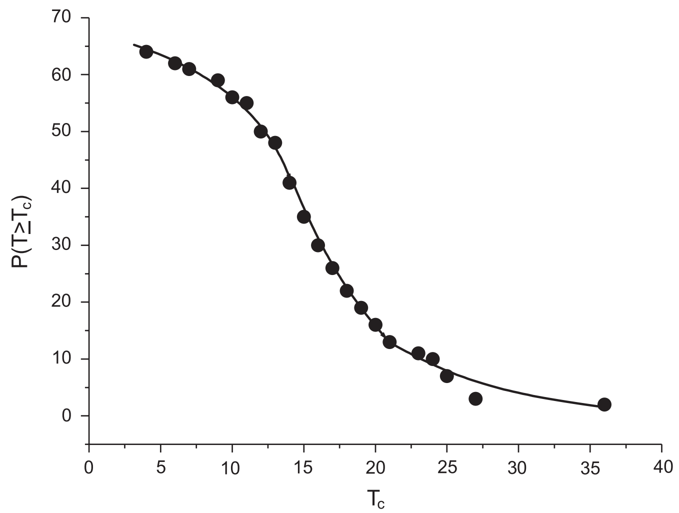

Fig. 1. Accumulative act-size distribution for UPTN-B.