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An evolutionary game theory model of binary opinion formation

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ABSTRACT

A basic characteristic of most opinion models is that people tend to agree or compromise in the opinion interaction, which could be hopefully described by cooperative games in the evolutionary game theory framework. This paper presents game theory methods to model the formation of binary opinions: cooperative games are proposed to model the interaction rules of general people who tend to find an agreement; minority games are proposed to model the behaviors of contrarians; opinion preference is considered by varying the payoff values. The Majority Voter model could be restored from the proposed games. The game theory models show evolutionary results similar to traditional opinion models. Specially, the evolution of opinions with consideration of contrarians is in accordance with the Galam model. Furthermore, influences of evolving rule, network topology and initial distribution of opinions are studied through numerical simulations. Discussions about methods to promote or hinder the consensus state at the best equilibrium point are given.

1. Introduction

Opinion Dynamics aims to understand how social opinions evolve and converge by defining different interaction mechanisms from individual levels [1,2]. From the aspect of how we define opinion variables mathematically, there are continuous opinion models [3–7] and discrete opinion models [8–16]. In continuous opinion models, opinion variables are usually real-valued scalars within a given interval, or sometimes real-valued vectors [7]. These models could help to understand the evolution of opinion in social scenarios such as the decision-making of train fare in public hearing. In discrete opinion models, opinions could be some given values. Typical discrete opinion models are Sznajd model [8,9], Galam model [10,11], Majority Voter model [12,13], discrete Deffuant model [14], discrete CODA model [15], etc. The most studied discrete opinion model is binary opinions or binary choices, since it could be considered simply to give an *yes/no*, or *agree/disagree* to any proposal or suggestion. In Ref. [16], the author proposed a unifying stochastic frame to classify the existing binary opinion models, showing that different local update rules may yield the same model and content. It is worth noting that *accept/reject* in diffusion models [17,18], *infected/susceptible* in epidemics [19], as well as *cooperate/defect* in cooperative games [20] could also be understood as binary strategies or binary opinions.

Either continuous or discrete, the general framework of opinion models is as follows: A distribution (usually chaotic) of opinions exists in a given society; at each time step, some groups of individuals interact according to some dynamical rule, and they generally tend to find an agreement or compromise among the group; due to these interactions, people finally reach a consensus on a certain opinion or form some opinion clusters (a cluster is a group of people holding the same opinion). The hypotheses of opinion models imply such a general fact that people tend to cooperate while exchanging their opinions, and these interactions cause opinions shift towards consensus among the whole society.

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Table 1
Game 1. Payoffs for agents.

	Opinion A	Opinion B
Opinion A	C	0
Opinion B	0	C

Cooperation is abundant in nature ranging from microbial interactions to human behaviors. The evolution of cooperation has long been a hot research topic in Biology, Economics, and Sociology [21–23]. Game theory and evolutionary game theory [22–24] are important tools in the study of cooperation. Researches are devoted to model various cooperative phenomenons [20,22,23], explain emergence and evolution of cooperation [21,25], and explore methods to promote cooperation [26,27]. In Ref. [28], the authors used methods from statistical physics to discuss the evolution of cooperation in the prisoner's dilemma, showing that there are intriguing interdisciplinary links between evolution game theory and physics. In this paper, we aim to adopt methods from evolutionary game theory to model opinion formation, and show interesting relations between binary opinion models and cooperative games.

Previous researches have witnessed some attempts to model the opinion formation using game theory methods. In Ref. [29], the authors modeled the interactions between individuals with one-shot games, with interaction strategies to be keep one's own opinion, agree with the other's opinion, and take a compromise opinion. They successfully obtained the famous Deffuant model [3,4]. On the basis of their work, we took consideration of a new 'not participate' strategy, and put the one-shot game into the evolutionary game theory framework [30]. Abundant dynamical regimes were obtained when we also considered strategy noise and preference. These works are within the framework of continuous Bounded Confidence model [5]. However, we find in these models that the confidence bound is very hard to explain. Besides, the definitions of interaction strategies and their payoffs are complex. In Ref. [31], the authors model the opinion interaction with a battle-of-the-sex game. According to their study, people would finally reach a consensus in the majority opinion or fall into fractions of co-existing opinions. In this paper, we model the opinion interaction more concisely using cooperative games and minority games. By setting the payoff values differently, opinion preference could be well simulated. We would also show how to obtain the Majority Voter model from the proposed model, and some dynamical results about how to influence the winning probability of the opinion with preference.

The paper is organized as follows. In Section 2, we propose some games to model binary opinion formation. Section 3 is devoted to discussions about evolutionary characteristics and implications of the model. In Section 4, we use numerical simulations to show influence factors of the best equilibrium state. Finally, we draw our conclusions in Section 5.

2. The games

As in most opinion models, we consider a group of agents joining in the opinion interaction, with edges representing interacting relations among them. Then, the social networks on which the opinion evolution takes place could be described as some certain complex networks, if we neglect evolution of network topology during the opinion interaction. Basically, most opinion models focus on developing some proper local dynamical rules from individual level, which would always lead to global evolution and synchronization. Here in this paper, we use evolutionary games to model the local dynamical rule, i.e., how agents rearrange their opinions through mutual discussions.

To build a game theory model, we need to specify players of the game, their optional strategies, payoff of each option, as well as evolving rules for evolutionary games [22–24]. We start by assuming that agents joining in the opinion interaction are players in games, and that these agents have equal opportunities to exchange their opinions with their directed linked neighbors in the interaction networks. For each time step, each agent updates the individual opinion through mutual discussions in the neighborhood, which are described by evolutionary games. As we have mentioned earlier, people in the opinion interaction tend to cooperate with others, i.e., to shift to the dominant opinion. Thus in the basic interaction model, we adopt a general cooperative game to describe such a tendency. For any agents in a mutual discussion, we assume that strategies are to hold opinion A or opinion B, which is also to say that *the evolution of strategies also stands for the evolution of opinions* (in the following part of the paper, we use 'opinion' to stand for 'strategy', and 'the evolution of opinions' to stand for 'the evolution of strategies'); cooperation brings more profit, which means that agents are better rewarded in the game if they hold the same opinion; the payoff value of cooperation is set to be C. The payoff matrix for each agent in a mutual discussion is as in Game 1 (Table 1). It should be noted that in the evolutionary game, every agent would coordinate with all his/her neighbors before making a strategic decision.

In Game 1, payoff values for both opinions are equal. But in social facts, opinion preferences are abundant. Either of the two opinions might be more attractive in social concept, religion, or pragmatistic belief. Here we consider the opinion preference by varying the payoff variables. For each agent, if one opinion is more attractive (A, for example), he/she would be better paid if chooses A. We suppose that the extra profit for the agent to choose opinion A is x. Then we get a model with opinion A preferred as in Game 2 (Table 2).

Until now, we have defined interaction games for general people who tend to cooperate with others. In social lives, contrarians are commonly seen [32], and they are playing an important role in public opinion formation [33,34]. In this paper, we model the interaction rule of contrarians also with an evolutionary game. We assume contrarians are those who