Tracking the maximum power transfer and loadability limit from sensitivities-based impedance matching

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ABSTRACT

This paper is concerned with voltage instability detection in power systems. The *Impedance Matching* (IM) equations are formulated as sensitivities-based equations and practical issues about impedance matching are addressed. This research presents a direct relationship between IM and power injection sensitivities, and proposes the use of a Sensitivity Impedance Matching–(SIM) for detection of instability conditions. SIM may detect the Maximum Transfer Point (MTP) and the Saddle-Node Bifurcation (SNB – the maximum loadability point). Prediction capabilities of SIM based on anticipation parameters and reactive generation limitation in advance are presented. Approximated active, reactive, and also apparent margin indices are proposed. The paper introduces novel theoretical results for designing reliable Voltage Instability Detection (VID) schemes.

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1. Introduction

Voltage instability is a potential source of power system blackouts [1,2]. Nowadays, it is important to develop novel applications and tools that allow monitoring the power system's operation in a reliable way, and that help to prevent severe conditions that may lead to voltage collapse.

The phasor measurement unit (PMU) technologies, together with advances in computational tools, networking infrastructure, and communications, have opened new perspectives for designing Voltage Instability Detection (VID) [3]. PMU-based voltage stability monitoring approaches can be classified into two broad categories: (*i*) those requiring observability of the whole region prone to voltage instability; (*ii*) methods based on local measurements, which can be implemented in a distributed manner [3]. Some examples of methods within the first category are those presented in [4–7]. In [4] and [5], an extended set of equilibrium equations are fitted to the available system state; then, an efficient sensitivity analysis is performed. The sensitivities of the total reactive power generation respect to individual loads [4], calculated from the fitted model, are used for voltage instability estimation. The method takes the effects of over-excitation limits (OELs) and load tap changers (LTCs) into account, and ignores the load dynamics. In [6], PMU's information

and the bus admittance matrix are used to evaluate the voltage stability at a bus and a voltage collapse prediction index is proposed. In [7] the sum of the absolute values of the complex voltage drops on the branches located on the shortest path from the load bus under analysis to the nearest generator under voltage control is assumed to be equivalent to the voltage drops across the Thévenin impedance. However, a full PMUs availability is required.

With some exceptions, for instance [8,9], most of the methods based on local measurements rely on the IM condition [10]. The basic assumption is that the whole power system analyzed from one load bus may be replaced by its equivalent. The IM is a promising tool for voltage instability detection (VID); this equivalent [11], and subsequent related contributions [12–17], have been proposed for on-line power systems control and protection, and several proximity indicators based on this concept have been developed [10,12–17].

Due to its simplicity, the IM condition has received great attention in the last two decades. However, some practical issues related to the IM are open. For instance, it is not clear the way a complex, nonlinear, discontinuous power system could be embedded into a simple equivalent, which is a linear circuit's notion [3]. Likewise, the IM requires system operating condition changes. The equivalent impedance has to be estimated from measurements gathered over a time window with two or more points that should be wide enough in order to the operating conditions may change, but narrow enough to satisfy the constant impedance assumption [3,11].

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Section 2 of this paper uses a particular case of IM, named *Local Impedance Matching* (LIM), to formulate a sensitivities-based set of equations for solving the IM problem that shows a direct relationship of IM with power injection sensitivities (dP_i/dV_i and dQ_i/dV_i). Based on this, some issues about the implementation of impedance matching for VID schemes are addressed. In Section 3, *Sensitivities-based Impedance Matching*–(SIM) are proposed; approximated active, reactive, and also apparent margin indices are proposed. Additionally, the use of SIM to detect and anticipate instability conditions is proposed. Finally, in Section 4, simulating results are presented that demonstrate the SIM ability to detect and anticipate instability conditions.

2. Theoretical background

The well-known equivalent equation at the *i*-th bus may be written as,

$$E_i = V_i + Z_{equ,i}I_i \tag{1}$$

where E_i is the equivalent voltage at the *i*-th bus; V_i is the *i*-th bus voltage; $Z_{equ,i}$ is the equivalent impedance at the *i*-th bus, and I_i is the corresponding injected current. On the Maximum Power Transfer Point (MTP), the magnitude of the equivalent impedance at the *i*-th bus ($Z_{equ,i}$) is equal to the magnitude of the load impedance ($Z_{Load,i}$). The impedance ratio is defined as [10],

$$k_i = \frac{||=Z_{equ,i}||}{||Z_{Load,i}||} \tag{2}$$

and is commonly used as a voltage instability index. For stable equilibrium $k_i < 1$; at the point of collapse (PoC) $k_i = 1$; and, for unstable equilibrium, $k_i > 1$.

In order to compute $Z_{equ,i}$, (1) may be rewritten to obtain the IM basic equations in matrix form as,

$$\begin{bmatrix} 1 & 0 & -i_{x,i} & i_{y,i} \\ 0 & 1 & -i_{y,i} & -i_{x,i} \end{bmatrix} \begin{bmatrix} E_{i,r} \\ E_{i,i} \\ R_{equ,i} \\ X_{equ,i} \end{bmatrix} = \begin{bmatrix} v_{x,i} \\ v_{y,i} \end{bmatrix} \tag{3}$$

where $E_i = E_{i,r} + jE_{i,i}$; $V_i = v_{x,i} + jv_{y,i}$; $I_i = i_{x,i} + ji_{y,i}$; $Z_{equ,i} = R_{eqi,i} + jX_{equi,i}$. Note that components $v_{x,i}$, $v_{y,i}$, $i_{x,i}$ and $i_{y,i}$ are directly available from measurements at the local bus. The unknown variables become the equivalents components $E_{i,r}$, $E_{i,i}$, $R_{equ,r}$, $X_{equ,i}$. Ordinarily, measurements taken at two or more different times are required to calculate them. The IM Eq. (3) has infinite solutions depending on the reference angle. Solution attains the equivalent Thévenin impedance when the reference angle becomes close to the angle of the Thévenin voltage [18]. Several methods have been proposed to solve (3). Conventional Least-Square (LS) [11], Recursive Least Squares (RLS) [14] and the Kalman Filter [19] have been used. In [15] the variables are estimated by an adaptive algorithm, assuming zero resistance. A method based on three successive PMU measurements is developed in [16]. Likewise, a nonlinear least squares strategy is used in [20]. Based on the Tellegen's theorem, in [13] it is shown that the solution of (3) becomes $Z_{equ,i} = \Delta V_i/\Delta I_i$, where the increments are calculated between two consecutive measurements. In [20], online load identification combined with the IM is proposed; different critical indications are derived for each load representation.

In [16], a preliminary study of the method called *Local Impedance Matching* (LIM) has been presented for detecting the MTP. The method is denoted by LIM, since it uses only measurements taken at the corresponding bus without needing a reference signal. Notice that the LIM is a particular case of the IM, where the bus voltage reference angle is taken at the own bus; that is, $V_i = ||V_i||\angle 0°$. The

LIM basic equations can be obtained multiplying (1) by voltage V_i. Thus, the next equation arises,

$$\begin{bmatrix} ||V_{i,t}|| & 0 & -P_{i,t} & -Q_{i,t} \\ 0 & ||V_{i,t}|| & Q_{i,t} & -P_{i,t} \end{bmatrix} \begin{bmatrix} E_{i,r} \\ E_{i,i} \\ R_{equ,i} \\ X_{equ,i} \end{bmatrix} = \begin{bmatrix} ||V_{i,t}||^2 \\ 0 \end{bmatrix} \tag{4}$$

where $S_i = V_iI_i^* = V_i(i_{x,i} - ji_{y,i}) = P_i - jQ_i$. Subscript t has been added in order to emphasize that information has been collected at time t. Thus, the IM equivalent impedance has to be estimated from measurements gathered over a time window with two or more points, which reflect a change in the operating condition. This represents a pragmatic problem: the minimum and maximum window width and its relationship respect to the change in the systems' conditions that lead to a valid IM equivalent estimation. Usually, the width of the measurement window is selected taken into account some empirical rules. However, it is not clear the way these rules may affect the results of the equivalent impedance identification. To address this issue, assume that system (4) is subject to any perturbation in $||V_{i,t}||$ $P_{i,t}$, and $Q_{i,t}$. The perturbed system may be represented by,

$$\begin{bmatrix} ||V_{i,t}|| & 0 & -P_{i,t} & -Q_{i,t} \\ 0 & ||V_{i,t}|| & Q_{i,t} & -P_{i,t} \end{bmatrix} \begin{bmatrix} E_{i,r} \\ E_{i,i} \\ R_{equ,i} \\ X_{equ,i} \end{bmatrix}$$

$$+ \begin{bmatrix} \Delta V_{i,t} & 0 & -\Delta P_{i,t} & -\Delta Q_{i,t} \\ 0 & \Delta V_{i,t} & \Delta Q_{i,t} & -\Delta P_{i,t} \end{bmatrix} \begin{bmatrix} E_{i,r} \\ E_{i,i} \\ R_{equ,i} \\ X_{equ,i} \end{bmatrix}$$

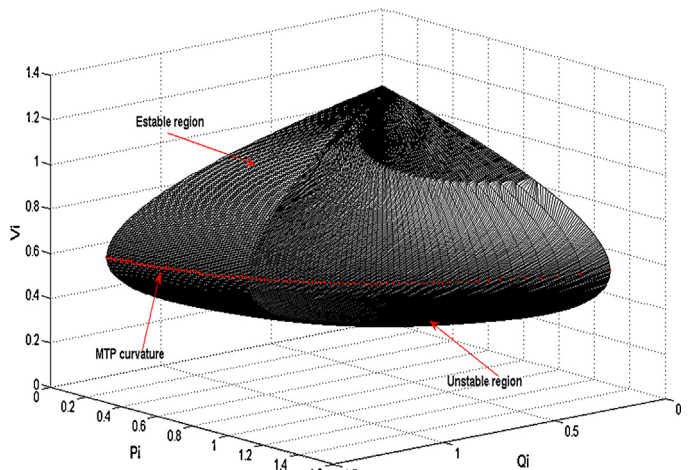$$= \begin{bmatrix} (||V_{i,t}|| + \Delta V_{i,t})^2 \\ 0 \end{bmatrix} \tag{5}$$

Fig. 1. PQV surface.
Adapted from [2].