Application of Artificial Bee Colony Algorithm to Maximum Likelihood DOA Estimation

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Abstract

Maximum Likelihood (ML) method has an excellent performance for Direction-Of-Arrival (DOA) estimation, but a multidimensional nonlinear solution search is required which complicates the computation and prevents the method from practical use. To reduce the high computational burden of ML method and make it more suitable to engineering applications, we apply the Artificial Bee Colony (ABC) algorithm to maximize the likelihood function for DOA estimation. As a recently proposed bio-inspired computing algorithm, ABC algorithm is originally used to optimize multivariable functions by imitating the behavior of bee colony finding excellent nectar sources in the nature environment. It offers an excellent alternative to the conventional methods in ML-DOA estimation. The performance of ABC-based ML and other popular meta-heuristic-based ML methods for DOA estimation are compared for various scenarios of convergence, Signal-to-Noise Ratio (SNR), and number of iterations. The computation loads of ABC-based ML and the conventional ML methods for DOA estimation are also investigated. Simulation results demonstrate that the proposed ABC based method is more efficient in computation and statistical performance than other ML-based DOA estimation methods.

Keywords: DOA estimation, maximum likelihood, artificial bee colony algorithm, bio-inspired computing

1 Introduction

The estimation of Direction-Of-Arrival (DOA) is an important problem in array signal processing, which can be widely used in the areas of radar, sonar, seismology, wireless communication, *etc.*. It has attracted great amount of interest for decades, and many useful estimation methods have been proposed and analyzed, including the Maximum Likelihood (ML) methods[1], the Multiple Signal Classification (MUSIC) methods[2], the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT)[3], *etc.*. The ML method is an excellent statistically effective and robust estimation technique. Its performance is better than the subspace decomposition class methods such as MUSIC and ESPRIT, especially under the conditions of lower Signal-to-Noise Ratio (*SNR*) or smaller snapshot number. Furthermore, ML method can estimate the parameters effectively when the sources are coherent signals, in which condition the subspace decomposition class

methods will lose efficiency. While we can get the optimal DOA angles through the ML method theoretically, but the ML estimator requires the maximization of a nonlinear multimodal likelihood function. Since it requires multidimensional solution search which makes the operation much more complicated, the application of the ML method is restricted.

Due to the characteristics of the ML method, many alternative multidimensional searching methods have been proposed in the past decades to reduce the the computational complexity, such as the Alternating Projection (AP) method[4], Space-Alternating Generalized Expectation-maximization (SAGE) method[5], Method Of Direction Estimation (MODE)[6], MODEX[7], and modified MODEX[8], *etc.*. Unfortunately, these methods still have some drawbacks that restrict their applications. The AP method converts the multidimensional searching to unidimensional searching, but its convergence becomes rather slow when the source number increases. The SAGE method requires detailed knowledge of the

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response of the measurement system in order to return reliable and accurate estimates, and the computational complexity may still be high due to the iteration processes. Most of the MODE-based methods require the eigen decomposition of data covariance matrix and there is a threshold restricting all the MODE-based methods. When the *SNR* or snapshot number is below the threshold value, the performance of these methods decreases greatly.

Recently, bio-inspired computing methods have been used in ML-DOA estimation, including Genetic Algorithm (GA)[9,10], Particle Swarm Optimization (PSO)[11–13], Differential Evolution (DE) method[13], and Clone selection algorithm (CLONALG)[13], *etc.*. The GA is one of the most powerful and popular global optimization tools. However, its application is somewhat limited by the precocity and slow convergence. To improve the performance of GA, several modified GA methods[14,15] were proposed, but their performances are not satisfied either. With the development of bio-inspired computing, more and more new methods were proposed. Boccato *et al*. applied some new meta-heuristic methods to the ML-DOA estimation[13], including PSO, DE and CLONALG. They demonstrated that these methods are capable to estimate the DOAs according to the ML criterion. They also compared the performance of each method with other conventional methods. They found that most meta-heuristic methods were efficient for ML-DOA estimation, and performed better than other classes of methods especially in lower SNR conditions. Moreover, there are many other efficient bio-inspired computing methods proposed in recent years, and it is worthy to test the performance of these methods applied to ML-DOA estimation and find which one is better.

Artificial Bee Colony (ABC) algorithm[17–20] is a newly proposed bio-inspired optimization method which simulates the behavior of bee colony finding excellent nectar sources. The most outstanding advantage of ABC algorithm is that it makes global and local optimized searching in each iteration, which can increase the probability of finding the global optimal solution and avoid the local optimal solutions. These characteristics contribute to the improvement of convergence and the reduction of iteration for the application of ABC algorithm to the ML-DOA estimation. The ML-DOA estimation based on ABC algorithm was first mentioned in Ref. [21], but it did not prove the applicability and the

superiority of ABC algorithm when it was used in DOA estimation. In this work, we apply ABC algorithm to the ML-DOA estimation with a modified probability function of onlooker bees becoming to employed bees, and compare its performance with other popular bio-inspired computing methods mentioned in Ref. [13] to demonstrate its convergence property, accuracy and efficiency for solving the problem of DOA estimation.

The organization of this paper is as follows: Section **2** presents the data model and the ML estimator. Section **3** provides the description of ABC algorithm and the ABC-ML estimator. Section **4** gives the simulation results to demonstrate convergence property and statistical performance of ABC-ML estimator, and compares it with DE-ML, PSO-ML and CLONALG-ML estimator. Section **5** concludes the paper.

2 Data model and maximum likelihood estimation

2.1 Data model

We assume that N narrow-band far-field signal sources impinge on an antenna array with M ($M > N$) isotropic sensors. Under classical assumptions, the array output vector at sampling time-instant k can be written as

$$\boldsymbol{y}(k) = \boldsymbol{A}(\theta)\boldsymbol{s}(k) + \boldsymbol{v}(k), \ k = 1, 2, \cdots, K \qquad (1)$$

where $\boldsymbol{y}(k) \in \mathbb{C}^{M \times 1}$ represents the snapshot data vector, $\boldsymbol{s}(k) \in \mathbb{C}^{N \times 1}$ represents the unknown vector of signal waveforms, $\boldsymbol{v}(k) \in \mathbb{C}^{M \times 1}$ represents an additive noise data vector. K denotes the number of data samples (snapshots). The array transfer matrix $\boldsymbol{A}(\theta) \in \mathbb{C}^{M \times N}$ has the following special structure as

$$\boldsymbol{A}(\theta) = \begin{bmatrix} \boldsymbol{a}(\theta_1) & \boldsymbol{a}(\theta_2) & \cdots & \boldsymbol{a}(\theta_N) \end{bmatrix}, \qquad (2)$$

where $\left\{ \boldsymbol{a}(\theta_n) \right\}_{n=1}^{N} \in \mathbb{C}^{M \times 1}$ is called steering vector and $[\theta_1 \ \theta_2 \ \ldots \theta_N]^{\mathrm{T}}$ are the parameters of interest. The exact form of $\left\{ \boldsymbol{a}(\theta_n) \right\}_{n=1}^{N}$ depends on the position of the nodes in sensor network. For the Uniform Linear Array (ULA), the steering vector has the form as

$$\boldsymbol{a}(\theta_n) = \begin{bmatrix} 1 & \mathrm{e}^{-j\omega_0 \frac{d}{c}\sin\theta_n} & \cdots & \mathrm{e}^{-j\omega_0(M-1)\frac{d}{c}\sin\theta_n} \end{bmatrix}^{\mathrm{T}}, \quad (3)$$

where $\omega_0 = 2\pi c/\lambda$, c is wave propagation speed and λ is the wave-length. $(\bullet)^{\mathrm{T}}$ denotes the transpose of matrix