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Study of bottleneck effect at an emergency evacuation exit using cellular automata model, mean field approximation analysis, and game theory

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ABSTRACT

An improved cellular automaton model for pedestrian dynamics was established, where both static floor field and collision effect derived from game theory were considered. Several model parameters were carefully determined by previous studies. Results obtained through model-based simulation and analytical approach (derived from mean field approximation) proved that outflow rate from an evacuation exit, which is usually estimated using outflow coefficient in building codes in Japan, can be improved by placing an appropriate obstacle in front of the exit. This can reduce collision probability at the exit by increasing collisions around the obstacles ahead of the exit.

1. Introduction

Analysis of pedestrian dynamics to predict the time taken to evacuate an enclosed space or even a whole building is very important from an engineering viewpoint. With respect to pedestrian flow, there have been three main approaches as in physics to the studies concerning traffic flow: continuum models (e.g., Ref. [1]), social force models (e.g., Ref. [2]), and cellular automaton (CA) models (e.g., Refs. [3,4]). Continuum models view pedestrian flow as fluid flow, which is a macroscopic feature or Eulerian in scope. While the latter two, called microscopic models, view pedestrian flow as human granular dynamics from the Lagrangian viewpoint. CA models can be expressed by a set of rules described spatiotemporally that are more flexible, physically intuitive, and applicable to real problems than the other two.

This paper concerns an important engineering problem: a bottleneck situation, where many pedestrians rush to an evacuation exit causing a drastic breakdown of outflow flux from the exit owing to "human jam effect", sometimes referred to as the "human arch" effect. In Japan, for example, the building construction codes strictly regulate the minimum requirement of outflow rate per unit exit width and unit time, called the outflow coefficient, for evacuation exits.

We focus on bottleneck problems in traffic flows because they are theoretically relevant to game theory. In fact, Yamauchi et al. [5] and Nakata et al. [6] found that several social dilemma structures are represented by n-person Prisoner's Dilemma (n-PD) games in specified traffic flow phases at a bottleneck caused by a lane-closing section.

Incidentally, one of the social dilemma structures, in which agents compete for a finite resource such as traffic capacity, natural resources, environment, or "an evacuation exit", is categorized by the *ST* (saint & temptation) reciprocity game [7], where players can obtain a higher payoff using different strategies rather than the same strategy. In a typical 2×2 game (2-player and 2-strategy game), in which each player can adopt one of two strategies, either cooperation (C) or defection (D), players receive a reward (R) for mutual Cs and punishment (P) for mutual Ds. If one chooses C and the other chooses D, the player choosing D obtains a temptation (T) payoff, while the C player is labeled a saint (S). $R > P$ and $T > S$ are assumed.

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		E		
3	2	1	2	3
$2+\sqrt{2}$	$1+\sqrt{2}$	2	$1+\sqrt{2}$	$2+\sqrt{2}$
$1+2\sqrt{2}$	$2+\sqrt{2}$	3	$2+\sqrt{2}$	$1+2\sqrt{2}$

Fig. 1. Assumed coordinate system for static floor fields around an evacuation exit.

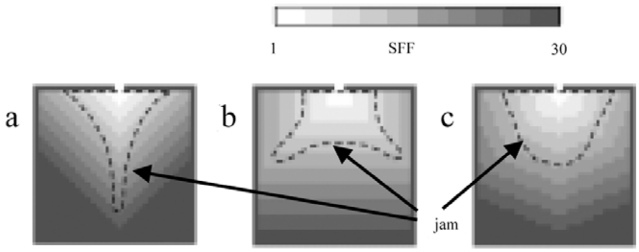

Fig. 2. Comparison of distribution of pedestrian numbers around an exit and its front line of jam (human arch). In the first coordinate system (a), agents cannot be allowed to move to askew four neighboring cells (they can move to von Neumann neighboring cells, in other words). In the second one (b), agents can move to Moore neighboring cells, but the askew potential difference on SFF is defined as 1, which is same as the horizontal and vertical differences. In the third one (c), which we finally adopted, agents can also move to Moore neighboring cells and the askew potential difference is defined as $\sqrt{2}$.

If $2R < S + T$ is valid in a game, agents are requested to establish *ST* reciprocity to avoid a dilemma-trapped situation. *ST* reciprocity might be relevant to our problem because a severe human arch occurs if pedestrians use the same strategy, "going ahead" simultaneously, while it does not occur if one tries to go ahead and the other yields.

Meanwhile, empirical and engineering knowledge indicate that the outflow rate from an exit is much better if appropriate obstacles are placed around the exit (e.g., Ref. [4]). At first glance, this seems contrary to common knowledge because an obstacle increases the flow rate rather than producing friction. We can now illustrate how this paradoxical phenomenon might be possible; since the obstacles make collisions happen before the exit, they can appropriately control the flow to avoid heavy jams at the exit, which is principally based on *ST* reciprocity.

This paper deals with the paradoxical phenomenon caused by the bottleneck effect, in which our approach of using CA simulation and mean field analysis dovetails with game theory. The CA model established is based on the static floor field (SFF) [4,8–13] and a stochastic concept related to statistical mechanics for analyzing the agent's dynamics.

2. Model

Our CA model, based on SFF [8–13], can take into account the collision effect by pedestrian agents, where model parameters are determined to reproduce the experimental data obtained by Yanagisawa et al. [12]. We know that several previous studies [8–11] applied not only SFF but also the dynamic floor field to emulate long-ranged and time dependent attractive interactions among pedestrians, in which an agent is trying to trace ruts left by preceding pedestrians similar to the pheromone in chemotaxis. In the present study, however, an assumed calculation domain, explained later, is simple, where a square (rectangular) room with a single evacuation exit, of which direction and location are known to all pedestrian agents in advance. Because of this, we had confirmed that SFF is enough to reproduce agents' evacuation plausibly.

2.1. Static floor field (SFF)

SFF defines the potential difference for agents to move on a floor. A room is divided into 0.5 m $\times$ 0.5 m cells, where several agents can never simultaneously occupy a single cell. As shown in Fig. 1, our SFF is assigned in a manner similar to that proposed by Zheng et al. [4], where diagonal differences are defined by $\pm\sqrt{2}$, while both horizontal and vertical differences are ± 1. This method seems appropriate because all agents can share the same SFF. However, Yanagisawa and Nishinari's [13] method, where, for example, $\sqrt{5}$ is allocated instead of $1 + \sqrt{2}$ in Fig. 1, cells cannot be shared with other agents because each agent must have a different SFF defined considering his standing position as the origin. This procedure inevitably requires a heavy computational load despite preferable accuracy from a geometric viewpoint. We confirmed that SFF can reproduce a realistic jam in front of an exit as shown in Fig. 2(c). For comparison, Fig. 2(a) is an alternative SFF where an agent is not allowed to move into four diagonal cells in the vicinity, and Fig. 2(b) is another system where an agent can move into four diagonal cells but its SFF difference is not $\sqrt{2}$ but 1.