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# Reliability optimization of a series system with multiple-choice and budget constraints using an efficient ant colony approach

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ABSTRACT

This paper deals with a reliability optimization problem for a series system with multiple-choice and budget constraints. The objective is to choose one technology for each subsystem in order to maximize the reliability of the whole system subject to the available budget. This problem is NP-hard and could be formulated as a binary integer programming problem with a nonlinear objective function. In this paper, an efficient ant colony optimization (ACO) approach is developed for the problem. In the approach, a solution is generated by an ant based on both pheromone trails modified by previous ants and heuristic information considered as a fuzzy set. Constructed solutions are not guaranteed to be feasible; consequently, applying an appropriate procedure, an infeasible solution is replaced by a feasible one. Then, feasible solutions are improved by a local search. The proposed approach is compared with the existing metaheuristic available in the literature. Computational results demonstrate that the approach serves to be a better performance for large problems.



## 1. Introduction

Reliability is a significant design measure in many industrial environments such as telecommunication systems and manufacturing facilities. The design of such hardware systems, called reliability optimization problem, can usually be based on either maximizing reliability, availability and performance, or minimizing cost. Reliability optimization of a series system has always been a critical matter. Subsystems of a series system are functionally organized such that any failure of each subsystem will cause the failure of the whole system. One of the strategies for increasing the system reliability of these sorts of systems is to use extra units in each subsystem in parallel. In this problem, reliability optimization is concerned with determining the optimal number of redundant units for one component employed in each subsystem. Many algorithms have been developed over the years to solve redundancy allocation problem (e.g. see Chen, 2006; Coit & Smith, 1996; Hsieh, 2003; Ramirez-Marquez & Coit, 2004; Ruan & Sun, 2006; Sung & Lee, 1994; Tavakkoli-Moghaddam et al., 2008; Yeh, 2009; You & Chen, 2005; Zhao & Liu, 2004; Zhao et al., 2007) and in some cases, reliability optimization is concerned with the design of *k*-out-of-*n* systems (e.g. see Tan, 2003; Yeh, 2004, 2006). As it is often desired to consider the practical design issue of handling a variety of different component types, this paper deals with a reliability optimization problem with multiple-choice constraints which has not received enough attention.

We consider a series system such that the reliability of the whole system should be maximized subject to multiple-choice and budget constraints. For each subsystem, a range of technologies is available among which only one must be chosen. If there is no constraint in the budget, then the most reliable technologies would be the most favorable. But, the available budget usually is limited and as the more reliable, the more expensive, a strategy is required to identify the optimal combination of technologies. This problem is called the reliability optimization of a series system with multiple-choice and budget constraints. The problem is formulated as a binary integer programming problem with a nonlinear objective function (Ait-Kadi & Nourelfath, 2001; Sung & Cho, 2000), which is equivalent to a knapsack problem with multiple-choice constraints, so that it belongs to the NP-hard class of problems (Garey & Johnson, 1979). Some exact algorithms have been developed to solve such knapsack problems with multiple-choice constraints (Nauss, 1978; Sinha & Zoltners, 1979) or the reliability problem (Sung & Cho, 2000) which are not efficient for large industrial problems because they require a very large amount of computation time to obtain the optimal solution. Therefore, the use of heuristics or metaheuristics is appeared to be necessary to attain optimal or nearly optimal solutions in a little time. Nourelfath and Nahas (2003) have proposed a heuristic approach based on the Hopfield model of neural networks. The approach applies a

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new model of Hopfield networks, where neurons take quantized values rather than just binary or continuous values. This heuristic is quickly able to obtain optimal or nearly optimal solutions of small problems. The first modern metaheuristic (and the only one based on our knowledge) has been proposed by Nahas and Nourelfath (2005) to solve the problem. In this algorithm, which is an ant system, called AS, a penalty treated in the pheromone trails update is employed for infeasible solutions concerning to the budget constraint. The penalties are proportional to the amount of budget violations. Also, a local search is applied to improve constructed solutions. The AS approach is quickly able to obtain optimal or nearly optimal solutions of large problems.

In this paper, we develop an efficient ant colony system, called ACS, for the problem. Ant colony optimization (ACO) (Dorigo, 1992; Dorigo et al., 1996; Dorigo & Stutzle, 2003) is a metaheuristic developed for solving discrete optimization problems. An ACO algorithm is a population-based approach based on the behavior of real ant colonies using pheromones as a communication medium. Real ants are capable of finding the shortest path from their nest to a food source without using visual cues. In the ACS approach, a solution is generated by applying a pseudo-stochastic rule based on a combination of the previous solutions results and the knowledge related to the problem as two fuzzy sets. The unfeasibility of constructed solutions is removed by replacing an infeasible solution by a feasible one based on a neighborhood search procedure. Each solution is then improved by an interesting local search. A set of large problems is used for evaluating the proposed approach.

The remainder of the paper is organized as follows. The next section gives the problem statement as a binary integer programming problem with a nonlinear objective function. The proposed ant colony approach is described in Section 3. Section 4 provides computational experiments and finally, concluding remarks are given in Section 5.

## 2. Problem formulation

Consider a series system that includes $S$ different subsystems. For subsystem $i$, there exist $N_i$ available technologies with different characteristics such as cost and reliability. Let $C_{ij}$ and $R_{ij}$ be, respectively, the cost and reliability of subsystem $i$ when technology $j$ is used. Total available amount of budget is $B$. The optimization problem is to choose only one technology for each subsystem to maximize reliability of the whole system ($R_{sys}$) subject to the available budget. In order to formulate the problem in mathematical expression, decision variable $x_{ij}$ is addressed as follows:

$$x_{ij} = \begin{cases} 1, & \text{if subsystem } i \text{ uses technology } j \\ 0, \text{otherwise} & \end{cases}$$

Then, the problem is formulated as the following binary integer programming problem with one nonlinear objective function:

$$\text{Max} \quad R_{sys} = \prod_{i=1}^{S}\left(\sum_{j=1}^{N_i} x_{ij}R_{ij}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{S}\sum_{j=1}^{N_i} x_{ij}C_{ij} \leqslant B \tag{1}$$

$$\sum_{j=1}^{N_i} x_{ij} = 1, \quad \forall\, i = 1, 2, \ldots, S \tag{2}$$

$$x_{ij} = \{0, 1\}, \quad \forall\, i = 1, 2, \ldots, S,\ j = 1, 2, \ldots, N_i \tag{3}$$

Constraint (1) represents the budget constraint, constraint (2) represents the multiple-choice constraint and constraint (3) defines the decision variables.

## 3. Proposed ant colony approach

In this paper, an ant colony system (Dorigo & Gambardella, 1997a, 1997b) based approach is developed for solving the reliability problem under consideration. To apply an ACO metaheuristic to a combinatorial optimization problem, it is appropriate to represent the problem by a graph $G = (\eta, ə)$, where $\eta$ and ə are, respectively, the nodes and edges. To represent the problem as such a graph, two types of nodes are introduced: the set of nodes $\eta_1$ containing one element for each subsystem and the set of nodes $\eta_2$ containing one element for each technology. Furthermore, the edges ə connect subsystems to their available technologies, that is, each node in $\eta_1$ is connected to each of the corresponding nodes in $\eta_2$ by an edge. In the proposed approach, an ant starts from the first subsystem and chooses (moves to) one of the available technologies for this subsystem. Then, the ant iteratively moves to the next subsystem and chooses a technology. At each step, a technology is chosen by applying a transition rule so-called pseudo-random proportional rule. Note that the generated solution may be infeasible; because constraints (2) and (3) are guaranteed during the construction process, but the total cost of the chosen technologies may be greater than $B$.

### 3.1. General structure of the approach

The general structure of the approach can be represented as follows (the next sections provide the details).

**Algorithm 1**

Step 1. The pheromone trails and the parameters are set.
Step 2. The following procedures are iterated *Max_iter* (an integer parameter) times:
  Step 2.1. The following actions are iterated *Ant_size* (an integer parameter) times:
    A. A solution is constructed by repeatedly applying the transition rule.
    B. If the solution is infeasible, it is replaced by a feasible one using Algorithm 2.
    C. If it is possible, the solution is improved by Algorithm 3, i.e., the local search procedure.
    D. The pheromone trails related to the chosen technologies are finally modified according to the local updating rule.
  Step 2.2. The pheromone trails are modified according to the global updating rule.
Step 3. The best solution found is printed.

### 3.2. Pseudo-random transition rule

Artificial ants probabilistically build solutions by iteratively choosing technologies by taking into account both the heuristic information on the problem and the (artificial) pheromone trails which change dynamically at run-time. An ant chooses one of the available technologies to assign to the current subsystem as follows: with probability $q_0$ an ant $k$ for subsystem $i$ selects the technology $j$ for which the product between the pheromone trail and the heuristic information is maximum, that is,

$$j = \arg\max[\tau_{ij}(\eta_{ij})^{\beta}] \tag{4}$$

where $\tau_{ij}$ and $\eta_{ij}$ are, respectively, the pheromone trail and heuristic information between subsystem $i$ and technology $j$ – denoted by edge $(i, j)$. Also, $\beta$ is a positive parameter denoting the relative importance of the heuristic information versus the pheromone trail.