European Journal of Operational Research 238 (2014) 699–710

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A multicut L-shaped based algorithm to solve a stochastic programming model for the mobile facility routing and scheduling problem

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ARTICLE INFO

Article history:
Received 5 June 2013
Accepted 21 April 2014
Available online 9 May 2014

Keywords:
Facilities planning and design
Routing
Fleet management
Mobile facility

ABSTRACT

This paper considers the mobile facility routing and scheduling problem with stochastic demand (MFRSPSD). The MFRSPSD simultaneously determines the route and schedule of a fleet of mobile facilities which serve customers with uncertain demand to minimize the total cost generated during the planning horizon. The problem is formulated as a two-stage stochastic programming model, in which the first stage decision deals with the temporal and spatial movement of MFs and the second stage handles how MFs serve customer demands. An algorithm based on the multicut version of the L-shaped method is proposed in which several lower bound inequalities are developed and incorporated into the master program. The computational results show that the algorithm yields a tighter lower bound and converges faster to the optimal solution. The result of a sensitivity analysis further indicates that in dealing with stochastic demand the two-stage stochastic programming approach has a distinctive advantage over the model considering only the average demand in terms of cost reduction.

1. Introduction

The Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP) are two well-studied problems in the field of operations research. The two problems are commonly coupled to deal with the process of delivery and pickup. For a conventional FLP, we usually consider facilities at fixed locations. The VRP is solved to handle the movement of items between facilities (e.g. depots or distribution centers) and customers (e.g. retailers). In reality, some vehicles behave in a way similar to traditional facilities when they are stationary except that they can move from one place to another if necessary. For instance, light trucks equipped with cellular base stations are able to provide cellular service to areas where the established cellular network temporarily fails (Halper & Raghavan, 2011); fast food service provided from vans can move to serve customers in different regions. In this paper, we use the term Mobile Facility (MF) to distinguish this “facility-like-vehicle” either from facility in the FLP or vehicle in the VRP. We define MF as a facility capable of moving from one place to another, providing certain real-time service to the customers in the vicinity of its location if and only if it is stationary.

The most distinct advantage of MFs over fixed facilities is their flexibility in moving to accommodate the change in the demand pattern over time and space. This can be illustrated with a simple example shown in Fig. 1. Given two customer points (A and B) and three time periods (t_1, t_2, t_3), the amount of demand for each customer in each period is shown below the node inside the squared box. Only one MF is available and the demand at A (or B) can only be served by locating the MF at A (or B). Additionally, it takes one time period to travel from A to B. With a fixed facility either at A or B, the total demand that can be served is 3 units (Fig. 1(b) and (c)). Note that the demand in A is decreasing while the demand in B is increasing. It is thus more beneficial if the MF can move from A to B during the second period, resulting in an increase to 5 units of the total demand covered (Fig. 1(d)). In a situation with multiple MFs, an efficient and effective tactical plan, including the route and time schedule, is needed in order to realize the perceived total benefits.

In the paper, we present a stochastic version of the mobile facility routing and scheduling problem by explicitly handling the stochastic demand, called as MFRSPSD. The goal is to decide the route and time schedule simultaneously for a fleet of MFs over a prespecified planning horizon, assign customer demands to MFs, and determine the amount of demand to be outsourced so as to minimize the total expected system-wide cost.

The remainder of the paper is organized as follows. Section 2 provides the problem description of the MFRSPSD, including the

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http://dx.doi.org/10.1016/j.ejor.2014.04.024
0377-2217/

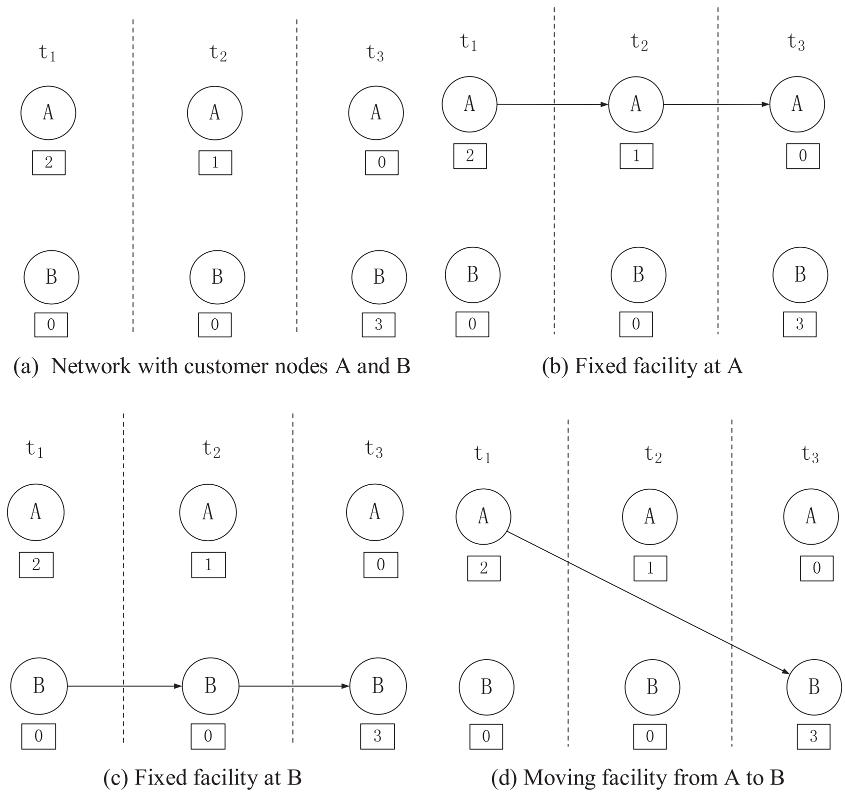

Fig. 1. Comparison of the performance of fixed and mobile facilities.

basic features, assumptions, and the expected input and output of the model. In Section 3, a detailed literature review is performed for research relevant to the MFRSPSD. A formal two-stage stochastic programming formulation of the MFSPSD is presented in Section 4 and a multicut L-shaped based method is developed in Section 5 to solve the problem. In Section 6, we present the computational results demonstrating the performance of the proposed algorithm. This is followed by Section 7 with conclusions and directions for future research.

2. Problem statement

The MFRSPSD is defined on a directed network $G(V,E)$ with node set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$. We denote the subset $I \subseteq V$ as the set of all customers points and the subset of nodes where MFs can be located as $J \subseteq V$. A distance matrix $D = (d_{ij})$ satisfying the triangle inequality is defined on E. The distances d_{ij} between any pair of nodes i and j are assumed to be deterministic and time-invariant. We assume that $d_{ij} = d_{ji}$, although it can be relaxed easily.

For simplicity in modeling, we divide the planning horizon into $|T|$ identical time periods, $T = \{t|t = 1, 2, \ldots, |T|\}$. Here the length of a time period is assumed to be sufficiently short such that, without loss of generality, all parameters within a time period are static. The demand level of each customer i for each time period t, denoted as $W_i^t(\omega)$, is time-dependent and stochastic which follow certain probability distributions, where ω represents a particular realization of the random variables. Let Ω be the set of all possible realizations of random variables, $\omega \in \Omega$.

Three important features of the MFRSPSD are considered: (a) the service equipment can be mounted on MFs which can move from one place to another, (b) the travel time of MF is explicitly accounted in the model and the service time are incurred only when MFs are not in motion and (c) the amount of demand to be served is proportional to the duration of the service time at the location serving the demand.

In addition, all MFs are assumed to be homogeneous providing the same service and traveling in the same speed. Denote the set of MFs as M, indexed by m. The travel time $T_{jj'}$ from location j to j' is assumed to be measured as an integer multiplier of a single time period. The travel time matrices are identical for all MFs due to the homogeneity assumption. Considering the expense of purchasing or renting an MF, staffing cost, equipment investment and so on, a fixed operating cost f needs to be paid for using an MF. Moreover, we impose a capacity limit C on the amount of demand that can be served by an MF in a single time unit. Thus, there is a possibility that the demand cannot be fully served by MFs due to the random fluctuation in demand over time and the limited capacity. For the demands that are not served, we assume that they are outsourced to a third party and a penalty cost $\gamma \geqslant 0$ is incurred for each unit of unmet demand. This penalty cost can either be considered as the opportunity cost for the loss of demands or regarded as an expense for outsourcing the excess demands to other companies.

Since an MF in motion is unable to provide service, it is undesirable to keep it moving for a long time, a potential loss of benefit. On the other hand, it is undesirable either to keep an MF stationary all the time, since it may lose the potential benefit resulting from making a strategic move to areas with more demands. Thus the major trade-off of the MFRSPSD involves the decision of moving or keeping stationary.

In some cases, moving would neither improve nor degrade the total performance. A traveling inconvenience cost α is thus introduced to discourage unnecessary moving in those cases. This is illustrated in Fig. 2. The setting of the example is similar to the one in Fig. 1 except that the demands are changed. In Fig. 2(a), the MF travels from A to B, and in Fig. 2(b), it stays at A all the time.