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Decision Support

A goal programming model for incomplete interval multiplicative preference relations and its application in group decision-making ☆

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ABSTRACT

In decision making problems, there may be the cases where the decision makers express their judgements by using preference relations with incomplete information. Then one of the key issues is how to estimate the missing preference values. In this paper, we introduce an incomplete interval multiplicative preference relation and give the definitions of consistent and acceptable incomplete ones, respectively. Based on the consistency property of interval multiplicative preference relations, a goal programming model is proposed to complement the acceptable incomplete one. A new algorithm of obtaining the priority vector from incomplete interval multiplicative preference relations is given. The goal programming model is further applied to group decision-making (GDM) where the experts evaluate their preferences as acceptable incomplete interval multiplicative preference relations. An interval weighted geometric averaging (IWGA) operator is proposed to aggregate individual preference relations into a social one. Furthermore, the social interval multiplicative preference relation owns acceptable consistency when every individual one is acceptably consistent. Two numerical examples are carried out to show the efficiency of the proposed goal programming model and the algorithms.

1. Introduction

In order to deal with a complex system of decision making, the analytic hierarchy process (AHP) was originally proposed by Saaty (1977, 1980) and it was further developed by Van Laarhoven and Pedrycz (1983) and Saaty and Vargas (1987), respectively. In the AHP, a decision-making problem is modelled as a hierarchy of criteria, subcriteria and alternatives. The decision maker (DM) may estimate his/her opinions over a set of alternatives by means of various preference relations, such as fuzzy preference relations, multiplicative preference relations, interval fuzzy preference relations and interval multiplicative preference relations and so on (see, e.g. Orlovski, 1978; Tanino, 1984; Arbel, 1989; Arbel and Vargas, 1993; Islam et al., 1997; Xu and Wei, 1999; Yager, 2004; Chandran et al., 2005; Herrera et al., 2005; Wang et al., 2005; Vaidya and Kumar, 2006; Wang and Elhag, 2007; Podinovski, 2007; Liu, 2009; Chiclana et al., 2009b; Prez et al., 2010; Siraj et al., 2012).

As shown in the above mentioned works, the preference relations with complete information are usually presented. To give a complete preference relation, one should make $n(n-1)/2$ judgments for a level with n criteria or alternatives. However, in a real decision-making problem, the experts may be lack of the knowledge of the problem and they may present incomplete preferences. That is, the DMs may express their judgments as incomplete preference relations in which some of the elements cannot be provided. Consequently, it is significant to provide the experts a tool to complement the lack of knowledge in their opinions. For example, Xu (2004) defined an incomplete fuzzy preference relation and proposed two goal programming models to obtain the priority vector from incomplete fuzzy preference relations. Xu (2006) further developed an algorithm to extend an incomplete multiplicative linguistic preference relation to a corresponding complete one, and gave a procedure to solve a GDM problem with incomplete multiplicative linguistic preference relations. A least-square method was developed by Gong (2008) to obtain the collective priority vector for GDM with incomplete fuzzy preference relations. Wang and Chen (2010) used incomplete fuzzy linguistic preference relations to ensure comparison consistency. Based on the additive consistency property, a model was proposed by Herrera-Viedma et al. (2007b) to estimate the missing information, then the GDM problems with incomplete fuzzy preference relations were dealt with. For an incomplete multiplicative preference relation, Fedrizzi and Giove (2007) gave a method of calculating its missing elements

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through minimizing a measure of global inconsistency. Chiclana et al. (2009a) showed that two different methods given by Herrera-Viedma et al. (2007b) and Fedrizzi and Giove (2007) are very similar in computing the missing values. Recently, Alonso et al. (2010) presented an implemented web consensus support system for GDM with incomplete information.

It is seen from the above stated works that the *consistency property* is always used to obtain a weight vector from an incomplete preference relation and a solution of a GDM problem with incomplete preference relations. When the decision makers express their opinions as incomplete interval multiplicative preference relations, it will be interesting and important to obtain a weight vector and a solution of a GDM problem. Based on the *consistency property* of interval multiplicative preference relations (Liu, 2009), this paper first give a goal programming model to estimate the missing values of the incomplete interval multiplicative preference relation. A solution of the model is obtained when the consistency level of the incomplete interval multiplicative preference relation reaches a maximum. This approach is different from those proposed by Herrera-Viedma et al. (2007a), Alonso et al. (2008, 2009b), and Chiclana et al. (2008). Additionally, a GDM problem with incomplete interval multiplicative preference relations is transformed into that with complete ones. An IWGA operator is proposed to obtain a social interval multiplicative preference relation. It is found that the social interval multiplicative preference relation is consistent or acceptably consistent when all individual ones are all consistent or acceptably consistent.

This paper is structured as follows. Section 2 introduces some known definitions and proposes the concepts of incomplete, consistent incomplete and acceptable incomplete interval multiplicative preference relations, respectively. In Section 3, a goal programming model is shown to complement an acceptable incomplete one, and a novel procedure is further given to obtain a priority vector from incomplete interval multiplicative preference relations. Section 4 addresses an acceptable consistency analysis of social interval multiplicative preference relations. Furthermore, a practical procedure for obtaining a solution of a GDM problem with incomplete interval multiplicative preference relations is presented. Finally, the main conclusions are shown in Section 5.

2. Preliminaries

In the AHP, a multiple criteria decision making problem is structured hierarchically at different levels, which contain a finite set of alternatives such as $X = \{x_1, x_2, \ldots, x_n\}$. The DM compares each pair of alternatives in X and provides a preference relation. In what follows, let us recall the definitions of various preference relations.

Definition 1 (Saaty, 1977). $A = (a_{ij})_{n\times n}$ is called a multiplicative preference relation, if the elements of A satisfy

$$a_{ij} = \frac{1}{a_{ji}}, \quad a_{ii} = 1, \quad a_{ij} \in R^+, \quad \forall i,j = 1,\ldots,n, \tag{1}$$

where a_{ij} represents a multiplicative preference degree of alternative x_i over x_j.

Hereafter it is assumed that a_{ij} is measured by using the $1-9$ ratio scale (Saaty, 1977). Especially $a_{ij} = 1/9$ indicates that x_j is absolutely preferred to x_i; $a_{ij} = 1$ implies that there is no difference between x_i and x_j; $a_{ij} = 9$ means that x_i is absolutely preferred to x_j.

Definition 2 (Saaty, 1977). Let $A = (a_{ij})_{n\times n}$ be a multiplicative preference relation. If the following transitivity is satisfied

$$a_{ij} = a_{ik}a_{kj}, \quad \forall i,j,k = 1,2,\ldots,n, \tag{2}$$

then A is called a consistent multiplicative preference relation.

Moreover, using the $1-9$ ratio scale, Eq. (2) is equivalent to

$$\log_9 a_{ij} = \log_9 a_{ik} + \log_9 a_{kj}, \quad \forall i,j,k = 1,2,\ldots,n. \tag{3}$$

For an inconsistent multiplicative preference relation A, Saaty (1977) gave a CI and a CR to measure the level of inconsistency. That is

$$CI = \frac{\lambda_{max} - n}{n-1}, \quad CR = \frac{CI}{RI}, \tag{4}$$

where λ_{max} and n are the largest eigenvalue and the order of A, respectively. RI is a random index dependent on the orders of the matrices, and it is the average CI of a large number of randomly generated multiplicative preference relations. If a multiplicative preference relation A satisfies $CR \leqslant 0.10$, then A is considered to be acceptably consistent. While $CR > 0.10$, A is considered to be unacceptably consistent and should be adjusted to that with acceptable consistency (Xu and Wei, 1999).

Definition 3 (Saaty and Vargas, 1987). An interval multiplicative preference relation U is represented by

$$U = (u_{ij})_{n\times n} = \begin{bmatrix} [1,1] & [u_{12}^-, u_{12}^+] & \cdots & [u_{1n}^-, u_{1n}^+] \\ [u_{21}^-, u_{21}^+] & [1,1] & \cdots & [u_{2n}^-, u_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [u_{n1}^-, u_{n1}^+] & [u_{n2}^-, u_{n2}^+] & \cdots & [1,1] \end{bmatrix}, \tag{5}$$

where u_{ij}^- and u_{ij}^+ are non-negative real numbers, $u_{ij}^- \leqslant u_{ij}^+$, $u_{ij}^- = 1/u_{ji}^+$ and $u_{ij}^+ = 1/u_{ji}^-$. u_{ij} indicates that x_i is between u_{ij}^- and u_{ij}^+ times as important as x_j.

Furthermore, Liu (2009) has addressed the consistency and acceptable consistency of an interval multiplicative preference relation U. That is, letting $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ where

$$c_{ij} = \begin{cases} u_{ij}^+, & i<j, \\ 1, & i=j \\ u_{ij}^-, & i>j, \end{cases} \quad d_{ij} = \begin{cases} u_{ij}^-, & i<j, \\ 1, & i=j, \\ u_{ij}^+, & i>j, \end{cases} \tag{6}$$

one has two definitions as follows:

Definition 4 (Liu, 2009). Let U be an interval multiplicative preference relation. If the multiplicative preference relations C and D determined by using (6) are all consistent, then U is said to be consistent.

Definition 5 (Liu, 2009). Suppose that U is an interval multiplicative preference relation. If the multiplicative preference relations C and D determined by utilizing (6) are all of acceptable consistency, then U is acceptably consistent. Otherwise, U is said to be unacceptably consistent.

On the other hand, to give a complete interval multiplicative preference relation, the DM should make $n(n-1)/2$ judgments for a level with n criteria or alternatives. However, for a certain uncertainty involved in real decision problems, the DM may give an interval multiplicative preference relation with incomplete information. In what follows, we give the corresponding definition:

Definition 6. Let $U = (u_{ij})_{n\times n}$ be an interval multiplicative preference relation. U is called an incomplete one, if some of its elements can not be given by the decision maker, and the other given elements satisfy

$$u_{ij}^- u_{ji}^+ = 1, \quad u_{ij}^+ u_{ji}^- = 1, \quad u_{ij}^- \leqslant u_{ij}^+.$$

Moreover, using Definition 4 and Eq. (6), one has