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Discrete Optimization

Integer linear programming models for grid-based light post location problem

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ABSTRACT

Selecting optimal location is a key decision problem in business and engineering. This research focuses to develop mathematical models for a special type of location problems called grid-based location problems. It uses a real-world problem of placing lights in a park to minimize the amount of darkness and excess supply. The non-linear nature of the supply function (arising from the light physics) and heterogeneous demand distribution make this decision problem truly intractable to solve. We develop ILP models that are designed to provide the optimal solution for the light post problem: the total number of light posts, the location of each light post, and their capacities (i.e., brightness). Finally, the ILP models are implemented within a standard modeling language and solved with the CPLEX solver. Results show that the ILP models are quite efficient in solving moderately sized problems with a very small optimality gap.

1. Introduction

Many real-world facility location problems can be approximated by a grid-based system of small-sized cells. These cells can then be used to model a heterogeneous demand distribution. We can also express the amount of supply in each cell from its supply distribution relationships with the various potential facility locations. Based on these demand distributions and supply relationships, we can then determine the optimal capacities and locations to place our facilities while fulfilling certain objectives. In this paper, these types of location problems are referred to as *grid-based location problems* (GBLPs). In the GBLPs, we will seek the optimum number, location(s), and size(s) of facilities to place. The applications of GBLPs are wide ranging, and include problems in business, every discipline of engineering, defence, resource exploitation, and even the medical sciences. To make such complex decisions, we need to develop mathematical models, and procedures to solve them.

Determining optimal location is a common, and often complex, problem in business and engineering. Over the last several decades and especially in recent years, several methods have been developed in the area of location theory resulting in a number of notable solving methods. These methods are problem specific and particularly designed for the various types of the location problem. One of the most significant facility location problems was first proposed by Cooper (1963), now well-known as the multisource Weber problem. The Weber problem has a known number of facilities and all the fixed costs for the facilities are equal. Since 1963, a lot of research has been done on the Weber problem. Wesolowsky (1972) proposed a model for the solution of the Weber problem using rectilinear distances. Sherali and Nordai (1988) focused on a capacitated multi-facility Weber problem (CMFWP) and demonstrated that the CMFWP is NP-Hard. Manzour-al-Ajdad et al. (2012) proposed an algorithm for solving a single-source CMFWP. Katz and Cooper (1974) first proposed a probabilistic multi-facility Weber problem which was later revisited by Altinel et al. (2009). A two-dimensional facility model is discussed by Francis (1964) to locate multiple new facilities with respect to existing facilities.

The complexity of the location problem depends on the nature of the problem and the criterion to be considered to make the decision. These criterions are selected by the decision maker from the problem description (Teixeira and Antunes, 2008). Marín (2011) described a new discrete location problem where the number of customers allocated to every plant has to be balanced. Ingolfsson et al. (2008) described an ambulance location optimization model that minimizes the number of ambulances needed to provide a specified service level. The model measures service level as the fraction of calls reached within a given time standard and considers response time to be composed of a random delay (prior to travel to the scene) plus a random travel time. Drezner and Wesolowsky (1997) proposed a method of placing signal detectors to cover a certain area such that the probability that an event is not detected is minimized.

Another special type of location problem is a location problem with the objective of coverage, which is first introduced by Church

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and ReVelle (1974). It ensures a set of facilities for each customer. The key applications of this model are to find optimum location of emergency services, retail facilities, cell-phone towers and sensor networks. The well-known *uncapacitated facility location* (UFL) problem is similar to these problems, except for the consideration of variable transportation costs (Wolsey, 1998). Moreover, the UFL becomes a *capacitated facility location* (CFL) problem when there is an upper limit for the amount of supply (Ghiani et al., 2002; Chen et al., 2010). For more information on the coverage location models, readers are referred to Berman et al. (2010). Of all the models developed, it is important to recognize that models to solve the location problems can be classified into two distinct groups: discrete location analysis and continuous location analysis. Discrete location analysis, the most common form of modeling a location problem, typically refers to the use of a node-and-network (transportation) approach where facilities and supply points are modeled as the vertices and nodes (Domschke and Krispin, 1997). Continuous location analysis involves the modeling of the location problem on a continuous plane. With the continuous location–allocation problem, all customer demands are coordinate points and furthermore, the feasible solution for the optimal placement of the facilities can be any coordinate point in the plane. Daskin (1995) points out that modeling the location as a grid can be NP-Complete, and as such, a transportation network (discrete location analysis) is typically employed.

To make location decisions, we need to develop mathematical models, more specifically, *integer linear programming* (ILP) problems. To model a real-world problem, we generally need to consider a large number of discrete variables, a heterogeneous demand distribution, non-linear supply distributions, and fixed costs associated with facility placement. Furthermore, to get an optimum decision, the ILP models need to be designed in such a way that they will simultaneously determine the locations, sizes and number of facilities to achieve certain objectives. Combinations of these considerations make the problem a large scale ILP problem, which are generally not scalable and often become intractable even with small problems. Therefore different types of heuristics are used to find the near optimal solution. Genetic search algorithm has been used to find solutions for location problems (Abdinnour-Helm and Venkataramanan, 1998). A genetic search algorithm is also used by Taniguchi et al. (1999) to obtain a near optimal solution for a logistics terminal location problem that also factors in traffic conditions by using queuing theory and nonlinear programming to trade-off between both transportation and facility costs at terminals to minimize total logistics costs. In Aytug and Saydam (2002), a *genetic algorithm* (GA) is used to solve large-scale maximum expected covering location problems. Methods such as the simultaneous perturbation stochastic approximation (SPSA), finite difference gradient (FDG), and very fast simulated annealing (VFSA) algorithms have also been used. Bangerth et al. (2006) compared and analyzed the efficiency, effectiveness, and reliability of these optimization algorithms for solving location problems. They found that none of these algorithms guarantees the optimal solution, but demonstrated that both SPSA and VFSA are very efficient in finding nearly optimal solutions with a high probability. Other methods for solving location problems proposed in the past few years have included methods such as a gravity model (Kubis and Hartmann, 2007), ILP-based formulations (Chen et al., 2005), the use of a Tabu search (Gendron et al., 2003), and the usage of a Greedy Algorithm (Zhang, 2006). Canbolat and Wesolowsky (2010) proposed an alternate local search heuristic approach to solving the Weber problem with a probabilistic line barrier method.

In this paper, we propose new formulations for a multisource location problem with the goal of determining the optimal combination for a facility distribution problem: the number of facilities, the location of each facility, and their capacities. Furthermore, in real-world situations, demand is not a singular point, but rather, many individual points located adjacent to each other forming a heterogeneous distribution that is extremely complex in nature. Such facility location problems can be approximated by a GBLP, where the entire area of this location problem is divided into small cells. These cells are then used to locate the heterogeneous demand distribution. On the other hand, we can express the amount of supply in each cell associated with each individual facility located in a specific cell from its supply distribution relationship. From this demand distribution and supply relationship, we have to place sufficiently-sized facilities in such a way that we can fulfill certain objectives. Our research herein focuses on the development of ILP models for GBLP, using the grid-based light post location problem to make optimal decisions in installation of lights in a city park.

The remainder of this paper is organized as follows. Section 2 provides the description of the problem with demand and supply calculation process. In Section 3, we discuss the basic model with simplified supply distribution. In Section 4, we propose two enhanced models with enhanced supply distribution. Section 5 describes the result analysis. Section 6 ends with conclusions and future research opportunities.

2. Problem description

Suppose we consider a city park, described as a 2-dimensional grid of known dimensions. Light posts must be installed throughout the park to provide adequate lighting conditions. We must determine the location and light intensity of each light post such that dark areas are lit and excess (waste) lighting is minimized. The brighter the light source, the more expensive this will be due to installation and electricity costs. As such, the objective is to satisfy the demand as much as possible while minimizing excess supply. Factors affecting the number of lights, their size, and their placement are many and varied. In a city park, there are different areas used for various purposes. Trees in the park and its topography create demand variation throughout the park. Furthermore, installing lights in boundary regions would not be feasible due to various physical restrictions such as roadways and underground power cables for utility service. This city park can be represented by a grid-based area, where the heterogeneous demand distribution can be represented by each cell in the grid. The idea is that the light sources should be placed in such a way that the areas they illuminate don't overlap too much, but not so far apart that there are unlit cells.

2.1. Demand distribution

A key characteristic of this problem is that the demand distribution is not composed of discrete points but rather a gradient of interrelated demand points. While the exact relationship between the various points is either not easily defined or may not be known, the existence of a relationship can be assumed. For example, light level measurements can be taken at coarse discrete intervals in order to determine light requirements (demands). While the data is pertinent for discrete locations, the data does not imply that the area where no data has been taken has a value of zero. As such, in order to infill the data grid to estimate the light value, we can use the known data measurements to estimate the values for which no measurements were taken in the two-dimensional plane. To model the interrelated demand data (i.e., generate the demand distribution), we turn to *finite element method* (FEM), a technique commonly used in many fields of engineering (Chapra and Canale, 2002). We applied Liebmann's method (also known as the Gauss–Seidel method) to the demand grids. In order to apply the