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B-spline-based shape coding with accurate distortion measurement using analytical model

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ABSTRACT

In this paper, we present a new model to measure the contour point distortion for the vertex-based shape coding with B-splines, called accurate distortion measurement using analytical model (ADMAM). Different from existing distortion measurements containing approximation, quantization or parameterization process, our distortion is defined on the original B-spline. It is modeled as the shortest distance of associated contour point from the original B-spline, which is in line with the subjective-based objective quality metric. The geometric relationships are introduced to simplify the model computation, followed by a hybrid admissible distortion checking algorithm to reduce the execution time. Our theoretical analysis and experimental results demonstrate that the ADMAM can lead to the smallest bit-rate among all the distortion measurements that guarantee the admissible distortion, when the operational rate-distortion optimal shape coding framework is applied. Moreover, if the original contour has N_C points, it takes only $O(N_C)$ time for both peak and mean-squared segment distortion measuring paradigms, which is the lowest computational complexity among all the existing distortion measurements.

1. Introduction

Shape coding is the fundamental technology to implement the object-oriented video applications such as storage, retrieval, editing and interaction [1–7]. Among the existing shape coding techniques, the vertex-based shape coding with B-splines can reflect the local contour characteristics with nature reconstruction appearance, so it has unique advantages in the object-oriented video applications [8].

A typical vertex-based shape coding with B-splines consists of three components, including admissible control point band [9], distortion measurement [10–15], and control point encoding scheme [16,17]. Among them, distortion measurement plays a very important role in both rate-distortion performance and computational efficiency, since it highly relates to the actual reconstruction quality and consumes the computational time [18]. Before describing existing distortion measurement models, we provide the framework for the overall object quality assessment under the minimum–maximum (MINMAX) criterion, which is congruent with perceptual quality considerations [19].

Let $\mathbf{p}_0$, $\mathbf{p}_1$ and $\mathbf{p}_2$ be three consecutive control points which define a quadratic basis uniform non-rational B-spline segment of the approximate contour. This segment starts from the midpoint of the control edge $\mathbf{p}_0\mathbf{p}_1$ and terminates at the midpoint of $\mathbf{p}_1\mathbf{p}_2$ [2]. These midpoints are called *knots*. Let us also associate these knots with the two closest points on the original contour. Then, the overall object quality under the MINMAX criterion can be evaluated on the segment-by-segment basis. It corroborates the significance of the segment distortion measurement because it is the foundation of the overall reconstruction quality assessment. We describe them in Fig. 1. Now we review the existing segment distortion measurements and discuss their inherent limitations.

- *Polyline model* [2,3,10]: It inappropriately measures the distortion on the polyline approximated B-splines. Therefore it leads to many apparent erroneous, as highlighted by the rectangular boxes in Fig. 2(b).
- *Band model* [2,3,11,12]: It checks whether the quantized B-spline lies inside the band rather than checking whether the associated contour point upholds the admissible distortion. As a result, it ignores the sharp futures enclosed by the rectangular box in Fig. 2(c).
- *Parameterization model* [13,14]: It calculates the distortion as the distance between the associated contour point and the parameterized B-spline point. Although the admissible

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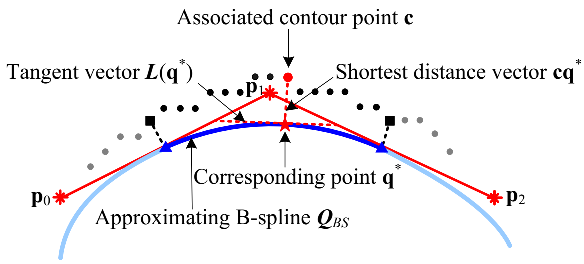

Fig. 1. The correspondence between B-spline segments and original contour segments [2], where our proposed ADMAM is defined. (Legend – *asterisk*: control point; *triangle*: knot; *square*: contour point associated with knot).

distortions are fulfilled, it overestimates the actual distortion and results in quite a number of extra control points and thereby additional encoding bits, as illustrated in Fig. 2(d).

The above problems motivate us to develop a novel model to measure the distortion, which should satisfy the following three requirements:

1. it can guarantee the admissible distortion,
2. the required bits should be as few as possible, and
3. the overall execution time should be as short as possible.

We find that the parameterization model is the closest one to satisfy all the requirements, except for requirement (2). So it is possible for us to construct a required model using the reference of this model. We find that this extra bits problem is due to the inappropriate B-spline point position produced by parameterization techniques. It may result in overestimated distortion values and discard the candidate approximating B-splines whose actual distortion can uphold the admissible distortion with fewer bits. This observation motivates us to construct a model that can accurately measure the actual distortion.[1] It has been reported that the actual distortion for reconstruction quality assessment is the minimal Euclidean distance between each associated contour point and the reconstruction contour [2,4]. Our distortion measurement can be modeled as follows.

Definition 1. The accurate distortion measurement for an associated contour point **c** is the shortest distance of **c** from the approximating B-spline Q_{BS}, as shown in Fig. 1. If we call this distortion as ADMAM, then it can be generally formulated as

$$ADMAM = \min_{\mathbf{q} \in Q_{BS}} \|\mathbf{cq}\|_2, \tag{1}$$

where

$$\mathbf{q}^* = \arg\min_{\mathbf{q} \in Q_{BS}} \|\mathbf{cq}\|_2, \tag{2}$$

is called the corresponding point, $\mathbf{cq}^*$ is called the shortest distance vector, and $\|\cdot\|_2$ denotes the L_2-norm.

Generally, Q_{BS} can be expressed as a parametric form given three consecutive control points. Thus, a straightforward method to compute (1) is to firstly find the explicit expression of ADMAM and then obtain its minimum value through mathematical analysis method. However, this method is quite complicated. Therefore, we consider it from the planar geometric aspect. We expect to find some geometric relation that can be described by simple formula.

Note that these geometric relations depend on three relative positions between **c** and Q_{BS}:

1. **c** is on the open Q_{BS} so $\mathbf{q}^*$ coincides with **c**,
2. **c** is off Q_{BS} and $\mathbf{q}^*$ is on the open Q_{BS}, and
3. **c** is off Q_{BS} and $\mathbf{q}^*$ is at the end of Q_{BS}.

Among them, case 2 is the most general one. Our investigation starts with case 2 and we expect that cases 1 and 3 can be integrated into the formula derived from case 2. The key observation is that for case 2, $\mathbf{cq}^* \perp L(\mathbf{q}^*)$, where $L(\mathbf{q}^*)$ is the *tangent vector* of Q_{BS} at $\mathbf{q}^*$ and $\perp$ denotes the perpendicularity. Thereby, the first two cases can be unified as a cubic equation:

$$\mathbf{cq}^* \cdot L(\mathbf{q}^*) = 0, \tag{3}$$

where $\cdot$ denotes the dot product, and for case 3, the boundary conditions can be added into the candidate parameter solution space to (3) to find the final ADMAM.

Since we calculate this distortion measurement by analytical method, we call it the *accurate distortion measurement using analytical model* (ADMAM). To further simplify its implementation, a hybrid admissible distortion checking algorithm is presented, based on the fact that the implementation of parameterization model is much simple and efficient and the distortion measurement calculated by parameterization model is always no less than ADMAM.

To sum up, the proposed ADMAM has the following four properties:

- it is in line with the subjective-based objective peak distortion assessment, so it is consistent and reliable,
- it totally avoids approximation, quantization or parameterization process on the original B-spline, therefore it is accurate and effective,
- when the vertex-based operational rate-distortion (ORD) optimal shape coding framework with B-splines under the MINMAX criterion [8,19] is exploited, it can guarantee the admissible distortion with the smallest bit-rate, and
- if the number of contour points is N_C, then it takes only $O(N_C)$ time for segment distortion calculation, which is the lowest complexity among the existing distortion measurements.

It is worth mentioning that although ADMAM is proposed for quadratic basis uniform non-rational B-splines; the main idea of our model and computational method can be used as reference model and computational method are also applicable to other types of curves, such as the recently proposed quasi-Bezier [21], the dynamic Bezier curve [22], and other high-order B-spline curve [23]. The main difference among them is that the derived parameter equations order will be higher. Therefore, the more sophisticated group theory needs to be applied for their solutions [24].

The rest of this paper is organized as follows. Section 2 elucidates an overview of the distortion measurements. Section 3 presents the results of existing geometric measurements, and provides the impetus for the development of a novel geometric distortion measurement. Section 4 describes the detailed ADMAM followed by its fast implementation. Full experimental analyses are provided in Section 5. Finally, some concluding remarks and future work are given in Section 6.

2. Existing segment distortion measurements: an overview

This section presents an overview of existing popular geometric distortion measurement for B-splines [18]. These mainly include three categories, polyline model based measurement, band model based measurement, and parameterization model based measurement. They can be incorporated into any vertex-based shape

[1] The preliminary idea behind this work was presented at IEEE International Conference on Image Processing (ICIP 2011) [20].