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Extended results on the relationship between information systems

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ABSTRACT

The relationship between information systems is an important topic in rough set field and it is studied through mappings. Many kinds of consistent functions have been proposed and they can be used to describe the invariance of attribute reducts between two information systems efficiently. However they cannot reflect relationships between neighborhoods generated from two information systems. In this paper, to achieve this aim, firstly, we introduce a neighborhood-continuous function that is inspired by the concept of continuous function in topology. Then, we address its properties and discuss relationships between neighborhood-continuous functions and several kinds of existing consistent functions. Furthermore, we investigate some important properties of neighborhood-continuous function with respect to relation mappings. Finally, based on neighborhood-continuous functions, the notion of a neighborhood-homomorphism between different information systems is proposed. It is noted that the reduct feature of a single information system can be described by neighborhood-continuous functions, while the reduct invariance of two different information systems can be described by neighborhood-homomorphisms.

1. Introduction

Rough set theory, proposed by Pawlak [15,16], is a useful mathematical tool for uncertain, inexact and fuzzy knowledge. It has been widely used in data analysis, data mining and artificial intelligence [9,11,13,20,29,31,37,38]. The original idea of rough set theory is based on an indiscernibility relation among elements of the universe. By using the indiscernibility relation, two definable sets are induced to approximate any given subset of the universe, called lower and upper approximations. The lower approximation is the greatest definable set contained in the given set, while the upper approximation is the smallest definable set containing the given set. Generally speaking, the upper approximation is larger than lower approximation. If the lower and upper approximations of a subset are just the same, then the subset is called a definable set; otherwise, called a rough set.

Rough set-based data analysis starts from a data table, which is also called an information system. Most studies of an information system focus on internal structures and properties of approximation spaces [1,3,9,14,18,21,30,33–35,40,45]. It is attractive that as a new research direction, the investigation on homomorphisms or mappings between information systems is receiving more attentions in recent years [6–8,12,19,23–27,36,39,42,43]. From mathematical viewpoint,

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homomorphisms or mappings can be considered as to compare structures and properties between different information systems, which have many important applications such as data compression, data fusion and information transmission [4,17,18,28]. The notion of homomorphism or mapping between information systems based on rough sets was first proposed by Grzymala-Busse [6]. In 1988, Grzymala-Busse depicted the conditions which make an information system to be selective in terms of endomorphism of the system [7]. Li et al. talked about the redundancy and reduct of information systems under homomorphisms or mappings [12]. Wang et al. investigated invariant properties of relation information systems under homomorphisms [22,24]. In order to disclose symmetric relationships between two kinds of important mappings proposed by Wang in [22], Zhu et al. introduced the notion of a neighborhood consistent function and unified Wang's mappings into neighborhood-consistent functions [41]. Homomorphisms or mappings cannot only reflect relationships between classical information systems, but also between more generalized information systems. For example, by using homomorphisms, Wang et al. presented some conditions under which attribute reducts of two covering-based information systems are equivalent to each other. Wang and Zhu discussed the properties of fuzzy information systems under homomorphisms [25,43]. Moreover, several invariant properties of ordered information systems and including degree-based information systems are systematically studied in [5,26,44].

A main contribution of these existing mappings was a description of invariance of attribute reducts in an original system and an image system. However, this kind of description is based on relation mappings with which a corresponding binary relation on one information system is induced by a binary relation on another information system. Therefore, if relations on both an original system and its image system have been given, these existing mappings cannot reflect relationships between the two binary relations. In this paper, in order to achieve this aim, we propose a neighborhood-continuous function that is inspired by the notion of continuous function in topology. We point out that neighborhood-continuous functions can unify some existing mappings between information systems and can compare two classes of granules between information systems. Moreover, based on neighborhood-continuous functions, the notion of a neighborhood-homomorphism between different information systems is proposed. We show that the notions of neighborhood-continuous functions and neighborhood-homomorphism are in accordance with structures of relation mappings. It is noted that the reduct feature of a single information system can be described by neighborhood-continuous functions, while the reduct invariance of different information systems can be described by neighborhood-homomorphisms.

The remainder of this paper is organized as follows. In Section 2, we review some relevant concepts about information systems and several kinds of mappings between information systems. Besides, we present definitions of continuous functions in topology. In Section 3, we introduce a neighborhood-continuous function and study its properties. In Section 4, we investigate the relationship between neighborhood-continuous functions and several existing consistent functions. Section 5 addresses some extended properties about relation mappings and studies neighborhood-continuous functions with respect to relation mappings. The results indicate that the notion of neighborhood-continuous function is in accordance with the structure of relation mapping. In Section 6, we describe the reduct feature of a single information system by using neighborhood-continuous functions and present the reduct invariance of different information systems by using neighborhood-homomorphisms.

2. Preliminaries

This section reviews some basic concepts related to this paper.

2.1. Basic concepts in information systems

The notion of a binary relation plays a basic role in information systems. Generally, for an arbitrary binary relation R_U on an object set U, R_U is called:

(1) reflexive if $(x,x) \in R_U$ for all $x \in U$;
(2) symmetric if $(x,y) \in R_U$ implies $(y,x) \in R_U$ for all $x,y \in U$;
(3) anti-symmetric if $(x,y) \in R_U$ and $(y,x) \in R_U$ imply $x = y$ for all $x,y \in U$;
(4) transitive if $(x,y) \in R_U$ and $(y,z) \in R_U$ imply $(x,z) \in R_U$ for all $x,y,z \in U$.

It is easy to see that relation R_U is an equivalence relation if and only if it is reflexive, symmetric and transitive simultaneously. Based on relation R_U on U, the predecessor and successor neighborhoods of each $x \in U$ can be defined as [33]:

$$R_{Up}(x) = \{y \in U | (y,x) \in R_U\};\ R_{Us}(x) = \{y \in U | (x,y) \in R_U\}.$$

An information system is a triple $\mathbf{S} = (U, A, V = \{V_a | a \in A\})$, where U is a nonempty and finite set of objects called the universe, $A = \{a_1, a_2, \ldots, a_n\}$ is a nonempty finite set of attributes such that $a : U \to V_a$ for any $a \in A$, i.e., $a(x) \in V_a, x \in U$, where V_a is called the domain for attribute a. If A is the union of two kinds of attributes, $A = C \cup D$, where C is the so-called condition attribute set, D is the so-called decision attribute set and $C \cap D = \emptyset$, then $\mathbf{S}$ is called a decision information system. For simplicity, we write an information system $\mathbf{S} = (U, A, V = \{V_a | a \in A\})$ as $\mathbf{S} = (U, A)$ [32,36].