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MOEA/D-ARA+SBX: A new multi-objective evolutionary algorithm based on decomposition with artificial raindrop algorithm and simulated binary crossover

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ABSTRACT

In the field of optimization computation, there has been a growing interest in applying intelligent algorithms to solve multi-objective optimization problems (MOPs). This paper focuses mainly on the multi-objective evolutionary algorithm based on decomposition, MOEA/D for short, which offers a practical general algorithmic framework of evolutionary multi-objective optimization, and has been achieved great success for a wide range of MOPs. Like most other algorithms, however, MOEA/D has its limitations, which are reflected in three aspects: the problem of balancing diversity and convergence, non-uniform distribution of the Pareto front (PF), and weak convergence of the algorithm. To alleviate these limitations, a new combination of the artificial raindrop algorithm (ARA) and a simulated binary crossover (SBX) operator is first integrated into the framework of MOEA/D to balance the convergence and diversity. Thus, our proposed approach is called MOEA/D with ARA and SBX (MOEA/D-ARA+SBX). On the other hand, the raindrop pool in ARA is further extended to an external elitist archive, which retains only non-dominated solutions and discards all others. In addition, the *k*-nearest neighbors approach is introduced to prune away redundant non-dominated solutions. In such a way, a Pareto approximate subset with good distribution to the true PF may be achieved. Based on the relevant mathematical theory and some assumptions, it is proven that MOEA/D-ARA+SBX can converge to the true PF with probability one. For performance evaluation and comparison purposes, the proposed approach was applied to 44 multi-objective test problems with all types of Pareto set shape, and compared with 16 other versions of MOEA/D. The experimental results indicate its advantages over other approaches.

1. Introduction

Many optimization problems encountered in the real-world frequently involve multiple conflicting objectives that should be optimized synchronously [1,2]. Such optimization problems are considered multi-objective optimization problems (MOPs), which can be defined as

$$\min_{\mathbf{x}\in\Omega} \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_k(\mathbf{x}))^{\mathrm{T}}, \tag{1}$$

where $\Omega = \prod_{i=1}^{D}[L_i, U_i] \subseteq \Re^D$ defines the feasible area, $\mathbf{x} = (x_1, x_2, \cdots, x_D)$ is the decision vector, D is the dimension of the decision vector, and L_i and U_i are the lower and upper bounds of the ith variable x_i, respectively. $\mathbf{F} : \Omega \to \Re^k$ consists of k real-valued objective functions and $\Re^k$ is called the objective space.

Because of the conflicting nature of the objectives, i.e., the amelioration of one objective gives rise to the deterioration in another, there is no single solution to optimize all objectives. Instead, the best trade-off solutions between different objectives are introduced in MOPs. In MOPs, a solution $\mathbf{x}_1$ is said to dominate $\mathbf{x}_2$ (denoted by $\mathbf{x}_1 \prec \mathbf{x}_2$) if and only if $\forall i \in \{1, 2, \cdots, k\}$, $f_i(\mathbf{x}_1) \le f_i(\mathbf{x}_2)$ and $\exists\, i_0 \in \{1, 2, \cdots, k\}$, and $f_{i_0}(\mathbf{x}_1) < f_{i_0}(\mathbf{x}_2)$. $\mathbf{x}^* \in \Omega$ is said to be *Pareto optimal* if no other feasible solution $\mathbf{x} \prec \mathbf{x}^*$ exists. The union of all $\mathbf{x}^*$ is termed a *Pareto set* (PS) and its image in the objective space is called *Pareto front* (PF). In addition, $\mathbf{x}^* \in \Omega$ is said to be a weakly Pareto optimal solution, if no other feasible solution $\mathbf{x}$ exists such that $f_i(\mathbf{x}) < f_i(\mathbf{x}^*)$ for all $i = 1, 2, \cdots, k$. It is noted that a Pareto optimal set is a subset of the weak Pareto optimal set.

Unlike single objective optimization problems, the PS of most MOPs is frequently composed of many or even an infinite number of solutions. Handing an unduly large number of Pareto

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optimal solutions is very time-consuming for the decision maker. In practice, most multi-objective evolutionary algorithms (MOEAs) are usually aimed at finding a representative and manageable approximate subset of the entire PF. Since Schaffer proposed the vector evaluated genetic algorithm (VEGA) [3] in 1985, a large number of MOEAs have been proposed to achieve the goal more efficiently. Based on different selection strategies, they can be broadly divided into three categories: (i) Pareto domination-based MOEAs, where the selection operator is based on the Pareto dominance relations among solutions [4]; (ii) indicator-based MOEAs, which adopt the evaluation indicators to guide the selection [5]; and (iii) decomposition-based MOEAs, which decompose an MOP into a number of scalar optimization subproblems using conventional aggregation approaches, and optimize them in a cooperative manner [6].

This paper focuses mainly on the multi-objective evolutionary algorithm based on decomposition, MOEA/D for short [6]. MOEA/D offers a practical framework for MOEAs by optimizing a series of single subproblems in a collaborative manner, and has been demonstrated to be very promising for a wide range of MOPs. Like most other algorithms, however, MOEA/D has its limitations. (1) The well-distributed PF obtained by MOEA/D depends so much on setting the appropriate weight vectors, which may not always be an easy task, in particular for some MOPs with irregular PFs. (2) The Tchebycheff aggregation function approach used in MOEA/D cannot ensure a Pareto optimal solution for each weight vector. (3) MOEA/D cannot easily balance the diversity and convergence effectively. In order to reduce these limitations, a new combination of the recently developed artificial raindrop algorithm (ARA) [7,8] and classical simulated binary crossover (SBX) [4] operator is first introduced to balance the convergence and diversity of MOEA/D. At the same time, the raindrop pool (**RP**) in ARA is further extended to an external elitist archive, which is utilized to store the non-dominated solutions found by MOEA/D in the process of the optimization search. The main role of the external elitist archive is to ensure a Pareto optimal solution for each weight vector, and ameliorate the defect of the initial setting of weight vectors in MOEA/D to a certain degree. In addition, for the purpose of generating a well-distributed PF, the *k*-nearest neighbors approach for the diversity measure is introduced to prune away redundant non-dominated solutions.

The main contributions of this paper can be summarized as follows:

1) A new combination of ARA and SBX is first integrated into the framework of MOEA/D to balance the convergence and diversity. Thus, a new MOEA/D with ARA and SBX, MOEA/D-ARA+SBX for short, is proposed.
2) Based on the relevant mathematical theory and some assumptions, it is proven that the proposed MOEA/D-ARA+SBX can converge to the true PF with probability one.
3) Systematic experiments conducted to compare MOEA/D-ARA+SBX with 16 other variants of MOEA/D on 44 multi-objective test problems with all types of PS shape are described. The experimental results indicate that the overall performance of MOEA/D-ARA+SBX is better than or comparable to that of other methods.
4) The results of experiments to investigate the effectiveness of the external elitist archive mechanism for some MOPs with irregular PFs by constructing a type of MOP with a sharp peak and low PF are reported.

The rest of this paper is organized as follows. In Section 2, the original ARA is introduced, which provides the necessary background for understanding the rest of the paper. In Section 3, the related work is described. The proposed approach, MOEA/D-ARA+SBX, is elaborated in Section 4. In Section 5, the experimental results and an analysis of the results of our comparative study are presented, and in Section 6, the impact of external elitist archive on the search performance of MOEA/D-ARA+SBX for MOPs with a sharp peak and low tail PF is discussed. Finally, Section 7 concludes the paper and provides some possible paths for future research.

2. Artificial raindrop algorithm

The ARA is a population-based intelligence algorithm inspired by the natural rainfall process. It transplants the change process of a raindrop in the natural environment, which is used to derive effective mechanisms, into optimization algorithm design. Derived from the observation of the natural rainfall process, the mainly cyclical process of ARA can be summarized as six stages: raindrop generation process → raindrop descent process → raindrop collision process → raindrop flowing process → **RP** updating process → vapor updating process. The flow of the algorithm follows the close-loop journey of a selected, finite number of raindrops, and its core idea is to follow the raindrops to the situation where they occupy the lowest energy state in the largest number-**RP**. In ARA, the raindrops are considered as objects and their performance is evaluated by their corresponding altitude, and the location with the lowest elevation corresponds to an optimal solution. The simulation scene graph of ARA and its general framework are displayed in Fig. 1(a) and Fig. 1(b), respectively. Next, the process of ARA is presented in brief.

Like most meta-heuristic algorithms, ARA begins with an initial population produced by randomly N vapors in a search space, each vapor has a corresponding position defined as

$$\mathbf{Vapor}_i = (x_i^1, \cdots, x_i^d, \cdots, x_i^D), i = 1, 2, \cdots, N, \tag{2}$$

where N is the population size, D is the dimension of the problem, and x_i^d is the position of the ith vapor in the dth dimension.

At a specific iteration t, the raindrop's position is considered the geometric center of the ambient water vapor. As a result, the raindrop generation (φ_R^G) operator can be defined as

$$\mathbf{Raindrop}(t) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^1(t), \cdots, \frac{1}{N}\sum_{i=1}^{N} x_i^d(t), \cdots, \frac{1}{N}\sum_{i=1}^{N} x_i^D(t)\right). \tag{3}$$

When the influence of external factors is ignored, the $\mathbf{Raindrop}(t)$ can be considered to drop from the cloud to the ground through free-fall. This means that one component of $\mathbf{Raindrop}(t)$ may be changed and $\mathbf{Raindrop}(t)$ moves to a new position denoted by $\mathbf{New\_Raindrop}(t)$. More specifically, let $\mathbf{Raindrop}^{(d_i)}(t)$ be the position of $\mathbf{Raindrop}(t)$ in the d_ith dimension, in which d_i ($i = 1, 2, 3, 4$) is chosen arbitrarily from the set {1, 2, ⋯, D}. Here, $\mathbf{New\_Raindrop}^{(d_1)}(t)$ can be obtained by a linear combination of $\mathbf{Raindrop}^{(d_2)}(t)$, $\mathbf{Raindrop}^{(d_3)}(t)$ and $\mathbf{Raindrop}^{(d_4)}(t)$, and the other components in $\mathbf{New\_Raindrop}(t)$ are the same as in $\mathbf{Raindrop}(t)$. As a result, $\mathbf{New\_Raindrop}(t)$ can be obtained by the raindrop descent operator (φ_R^D) as

$$\begin{cases} \mathbf{New\_Raindrop}^{(d)}(t) = \mathbf{Raindrop}^{(d_2)}(t) + \phi \cdot (\mathbf{Raindrop}^{(d_3)}(t) \\ \quad - \mathbf{Raindrop}^{(d_4)})(t), \text{ if } d = d_1; \\ \mathbf{New\_Raindrop}^{(d)}(t) = \mathbf{Raindrop}^{(d)}(t), \ otherwise, \end{cases} \tag{4}$$

where ϕ is a random number in the range (-1, 1), $d = 1, 2, \cdots, D$.

When $\mathbf{New\_Raindrop}(t)$ contacts the ground, it will be split into a number of small raindrops. Then, these small raindrops ($\mathbf{Small\_Raindrop}_i(t)$, $i = 1, 2, \cdots, N$) fly in all directions. For this reason, $\mathbf{Small\_Raindrop}_i(t)$ can be defined by the raindrop