Applied Soft Computing 41 (2016) 400–417

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Memory-based adaptive partitioning (MAP) of search space for the enhancement of convergence in Pareto-based multi-objective evolutionary algorithms

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ARTICLE INFO

Article history:
Received 16 April 2015
Received in revised form
19 November 2015
Accepted 10 January 2016
Available online 28 January 2016

Keywords:
Multi-objective evolutionary algorithms
Memory-based adaptive partitioning
Convergence improvement

ABSTRACT

A new algorithm, dubbed memory-based adaptive partitioning (MAP) of search space, which is intended to provide a better accuracy/speed ratio in the convergence of multi-objective evolutionary algorithms (MOEAs) is presented in this work. This algorithm works by performing an adaptive-probabilistic refinement of the search space, with no aggregation in objective space. This work investigated the integration of MAP within the state-of-the-art fast and elitist non-dominated sorting genetic algorithm (NSGAII). Considerable improvements in convergence were achieved, in terms of both speed and accuracy. Results are provided for several commonly used constrained and unconstrained benchmark problems, and comparisons are made with standalone NSGAII and hybrid NSGAII-efficient local search (eLS).

1. Introduction

Real-world optimization problems usually involve several conflicting objectives. The main aim in multi-objective optimization is to select the best trade-offs among these conflicting objectives. Multi-objective optimization problems (MOPs) can be mathematically stated as,

$$\begin{aligned} &\text{Minimize} \quad F(x) = (f_1(x), \ldots, f_m(x)) \\ &\text{Subject to} \qquad x \in \Lambda \end{aligned} \tag{1}$$

where, m is the number of objective functions, $x \in \Lambda$ is the vector of decision variables, and $F(x)$ consists of m objective functions $f_i(x)$. Conflicting trade-offs occur when an improvement in one objective function mirrors the deterioration of another one. Given two solutions x_1 and x_2, the solution $x_1 \in \Lambda$ is said to dominate $x_2 \in \Lambda$ if and only if $f_i(x_1) \leq f_i(x_2)$ for all objective functions and $f_i(x_1) < f_i(x_2)$ on at least one objective function, and is designated by $x_1 \prec x_2$. The Pareto front comprises all the optimal solutions belonging to the set of non-dominated optimal solutions known as the Pareto Set (PS).

Multi-objective evolutionary algorithms (MOEAs) [1–3] are regarded as promising methods for dealing with MOPs, owing to their ability to generate a population of solutions for efficiently approximating the diverse set of optimal solutions in a single run. Traditionally, MOEAs use derivative-free search algorithms [4], adapted to situations with different degrees of complexity: non-linear, stiff, black-box, and multimodal. Indeed, due to their population-based trait, MOEAs provide a wide range of conflicting alternatives for design (regarding the Pareto front), equivalent to an appropriate decision-making plan.

Since MOEAs were first implemented, following Schaffer [5]'s development of the vector evaluated genetic algorithm (VEGA), they have undergone considerable improvements, gaining a high profile and prompting many discussions. An interesting survey undertaken by Zhou et al. [6] highlighted the extensive developments made to MOEA frameworks over the past 10 years, ranging from decomposition-based (MOEA/D), preference-based, indicator-based, memetic and co-evolutionary MOEAs, to MOEAs with specific search methods and metaheuristics. Two main currents can be identified, each meeting to a contemporary scientific and industrial need: (i) improvements in the convergence accuracy/speed ratio for MOEAs, through the development and integration of new and efficient metaheuristics [7–11]; and (ii) the processing of expensive MOPs [12–14], the aim is to reduce the high numbers of physical experiments or time-consuming simulations that are currently performed.

The present study concentrated on the first current; seeking to improve the accuracy/speed ratio in MOEAs by refining the search space with no aggregation in objective space and no deterioration in the formulation of MOPs. To this end, we developed a memory-based adaptive partitioning (MAP) algorithm, applicable to the structure of Pareto-based MOEAs. In the context of the present study, the term *speed* corresponded to the number of function evaluations required to achieve convergence toward an optimal Pareto front, in order to be able to compare different algorithms independently of the performance of the computing facilities used.

It should however be noted that in the two main fields of research mentioned above, more attention has been generally paid to behavior in the objective space, rather than in the search space. A rare exception to this general tendency is the use of harmony search algorithms in multi-objective optimization, despite their major drawbacks, e.g. the low convergence and diversity performance. For instance, Dai et al. [15] investigated a self-adaptive multi-objective harmony search (SAMOHS) algorithm based on harmony memory (HM) variance. The harmony search comprises a memory-based stochastic search technique, where each solution is expressed by a real vector, dubbed a harmony. The memory will be updated

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http://dx.doi.org/10.1016/j.asoc.2016.01.029
1568-4946/

Notations

Λ	search space
Ω	partitioning multiplier
$\Delta\tilde{x}_i$	step size for a given search space i
E_r	Pareto front improvement within the last r generations
d_i	Euclidian distance
$\underline{D}_i$	restricted search space i
HVI	hypervolume indicator
I_i	variable importance
IGD	inverted generational distance
I_H^-	hypervolume distance between the Pareto optimal front and an approximated Pareto front
IN_i	interval numbers relative to a same category of variables i
$\hat{k}$	partitioning degree
m	number of objective functions
$[M]$	memory matrix
N	population size
n_V	number of variables
p	statistical p-value
P	population
PF	Pareto front
PF^*	optimal Pareto front
PT	partitioning tendency
PV	partitioning vector
$\underline{s}_{i,j}$	solution belonging to a restricted search space $\underline{D}_i$
x_k	vector of decision variables
$\underline{x}_k$	vector of restricted decision variables
$\tilde{x}_i$	vector of congruent variables i
$\underline{\tilde{x}}_i$	vector of restricted congruent variables i
$\widetilde{\underline{x}}_i^+$	maximum value in a set of restricted congruent variables $\underline{\tilde{x}}_i$
$\widetilde{\underline{x}}_i^-$	minimum value in a set of restricted congruent variables $\underline{\tilde{x}}_i$
$[\underline{\boldsymbol{x}}]$	matrix of restricted solution vectors

Index and exponents

l	lower
u	upper

if a better newly generated harmony can be found. SAMOHS was reported as being able to overcome the well-known drawbacks of MOHS, by providing enhanced local and global search abilities, and by applying a novel self-adaptive mechanism to the search algorithm.

Much of the contemporary literature on MOEAs deals with aggregated objectives. An aggregation-based approach entails either the conversion of MOPs into Single-objective Optimization Problems (SOPs) or else the decomposition of MOPs into a number of scalar objective optimization problems (parallel SOPs). One of the reasons why aggregation-based approaches are so commonly used in MOEAs is that most ingenious search algorithms were initially invented within an SOP landscape. Based on the same conventional aggregation approach, other methods, such as MOEA/D [16], involve decomposing an MOP into a number of sub-problems (SOPs). Each SOP is then optimized in a collaborative manner, using information provided by its neighboring SOPs. Here, the objective for each sub-problem is the weighted aggregation of the individual objectives. MOEA/D is a recent and very efficient MOEA that has been successfully applied in a number of areas. Moreover, by transforming an MOP into a parallel set of SOPs, advanced local search metaheuristics such as iterative local search (ILS), greedy randomized adaptive search (GRASP) or tabu search can easily be incorporated [17]. MOEA/D is therefore suited to the incorporation of other local search methods and selection operators. For instance, the combination of a new bandit-based adaptive operator selection (AOS) in MOEA/D [18], and the new memetic MOEA/D framework proposed by Qi et al. [19] can be mentioned. AOS allows the automatic selection of appropriate operators in an online manner within an optimization process. In the new memetic MOEA/D framework [19], a novel selection operator was designed to overcome limitations related the established Tchebycheff metric widely used in the original MOEA/D, together with three local search methods (Swap, Lamda interchange, Single route 2-opt) in order to deal with the multi-objective vehicle routing problem with time windows (MO-VRPTW).

The focus of the present study, however, was neither to modify commonly used frameworks and their evolutionary operators done by Alberto et al. [20], or by Yevseyeva et al. [21] in their new approach to selection in MOEAs based on portfolio selection problem, for instance, nor to reorganize the objective space and objective functions, as in the case of MOEA/Ds or clustering in multi-objective particle swarm optimization [22,23]. Rather, we sought to highlight the importance of search space refinement for improving convergence in MOEAs by preserving MOPs in their original multi-objective formulation. This is why we did not investigate aggregation-based MOEA frameworks. It should also be noted that the aggregation of objectives results in a prior ordering of objectives, where realistic interactions among objectives and their importance are susceptible to misevaluation or loss.

Despite their impressive capabilities, EGO-based frameworks such as ParEGO [14], SMS-EGO [13] and MOEA/D-EGO [12] were not included in our comparisons, because these methods are primarily used to deal with computationally expensive MOPs or when the computational budget in terms of function evaluations is extremely limited. It should be noted that when MOPs involve time-consuming simulations, it is sometimes useful to provide model-assisted algorithms with high computational complexity, in order to restrict the number of function evaluations required. However, in the case of SMS-EGO, this can result in a considerable increase in calculation time (several hours for a test problem with 8 variables and 200 function evaluations). MOEA/D-EGO involves the decomposition of MOPs. It uses parallel computations, where the algorithm builds a Gaussian stochastic process model for each of the sub-problems and simultaneously optimizes their improvement. The Gaussian stochastic process model is one of the most popular methods for dealing with expensive problems.

In recent decades, special attention has been paid to hybrid and memetic MOEAs, all the while acknowledging that original evolutionary algorithms result in a lower level of accuracy for optimal solutions, compared with deterministic approaches. The general trend in the hybrid approach is either to combine global and local search patterns, or else to combine the search operators of different algorithms. The framework of memetic MOEAs incorporates local search methods, to speed up convergence and provide better accuracy, deeply motivated by varying applications from engineering to business, economics, and finance [24–26]. Since our focus was on improving the convergence accuracy/speed ratio in MOEAs, we looked at both hybrid and memetic MOEAs in order to conduct a fair comparison. It should, however, be admitted that if an efficient memetic or hybrid MOEA algorithm offers simultaneous improvements in both accuracy and speed, the latter may also be regarded as a potential solution for processing expensive MOPs.

The local search metaheuristics involved in hybrid MOEAs perform either neighborhood-based or directional local searches. A neighborhood-based approach provides perturbations around a given solution to improve convergence [9]; while with a directional local search, a search direction is determined either by running a sensitivity analysis of objective functions [8] or by using previously evaluated solutions in neighborhoods to progressively build an efficient search direction [7]. Additional function evaluations are required to explicitly determine gradient vectors in the objective space for sensitivity-based directional local searches.

The combination of evolutionary algorithms and local search methods can be made in two ways: only applying local search methods to the final solutions yielded by the evolutionary algorithms [11]; or implementing them as supplementary search engines within the main evolutionary algorithm framework [7,8,27]. Deb and Goel [11] applied a hill climbing local search method to the final solutions of evolutionary algorithms. Lara et al. [8], on the other hand, introduced a new gradient-free local search strategy, known as the Hill Climber with Sidestep (HCS), within the NSGAII and SPEA-2 state-of-the-art frameworks, creating a new memetic MOEA. This hybrid HCS-MOEA framework was shown to improve performances for unimodal functions such as CONV1, CONV2, and DTLZ2. The hill climber tries to improve convergence to the Pareto front, while the sidestepper provides a lateral search for a better solution spread. As HCS is gradient-free, it utilizes the randomly generated neighbors of a present solution within a given radius in the search space. The line search procedure for HCS is to minimize a convex single objective by quadratic polynomial fitting. Inspired by the HCS local search operator, Kim and Liou [7] introduced a novel directional local search operator for EMOAs, called efficient local search (eLS). This uses a hill climbing method to improve convergence, but does not adopt the sidestepper in order to reduce the number of additional function evaluations. eLS follows a similar line search procedure to HCS except that eLS minimizes the aggregated and normalized composite objective, instead of running separate line searches for each objective function. The algorithm also performs adaptive tuning of the two sensitive local search parameters: local search probability and neighborhood search radius.

The recent hybrid MOEA-HCS and MOEA-eLS are among the most promising metaheuristics for improving the accuracy/speed ratio in MOEAs. In the present study, however, we only included the hybrid eLS in our comparisons, owing to its memory-based peculiarity of performing directional local search through previously evaluated solutions in the neighborhood without sidestepping. This procedure is intended to considerably reduce the number of additional function evaluations.

In the following sections, we begin by introducing the MAP algorithm. The state-of-the-art fast and elitist non-sorting genetic algorithm (NSGAII) developed by Deb et al. [3] was chosen to show how MAP can be integrated within a conventional