Information Sciences 329 (2016) 236–255

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Preference-guided evolutionary algorithms for many-objective optimization

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ARTICLE INFO

Article history:
Received 4 December 2014
Revised 7 August 2015
Accepted 14 September 2015
Available online 25 September 2015

Keywords:
Multi-objective optimization
Many-objective optimization
Preference-based optimization
Evolutionary algorithms
Decision making
Reference point

ABSTRACT

This paper presents a technique that incorporates preference information within the framework of multi-objective evolutionary algorithms for the solution of many-objective optimization problems. The proposed approach employs a single reference point to express the preferences of a decision maker, and adaptively biases the search procedure toward the region of the Pareto-optimal front that best matches its expectations. Experimental results suggest that incorporating preferences within these algorithms leads to improvements in several quality criteria, and that the proposed approach is capable of yielding competitive results when compared against existing algorithms.

1. Introduction

Among the available methods for computing *Pareto-optimal* solutions for multiobjective optimization problems (MOPs) [5,33], evolutionary algorithms (EAs) [55] have received a large amount of attention from the research community. This is mainly due to the ability of multi-objective evolutionary algorithms (MOEAs) to tackle these problems regardless of the convexity, modality, and differentiability properties of the objective and constraint functions. Moreover, they are capable of returning multiple non-dominated solutions after a single execution. For this second reason, MOEAs have also largely adopted the *a posteriori* approach to the solution of MOPs, *i.e.*, generate a set of (possibly Pareto-optimal) points from which a decision maker (DM, as defined in Section 2.1) can afterward choose a preferred one [21].

Despite its success on real-world applications, state-of-the-art evolutionary algorithms did not at first perform very well in problems belonging to the *many-objective optimization* field [26], that is, problems with four or more objectives. The main difficulties faced in this case are: (i) it is harder to visualize the solutions in order to make a final choice; (ii) the number of points needed to approximate the efficient front[1] usually grows exponentially with the number of objectives; and (iii) the selection mechanisms usually employed in general-purpose EAs tend to fail as the number of objectives is increased, leading to difficulties in approximating Pareto-optimal points.

Most methods for adapting MOEAs to many-objective problems concentrate on the third issue, keeping the *a posteriori* approach. This strategy is, however, often inadequate for this class of problems, since a DM will eventually need to select a single

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[1] As explained in the next section, the terms *efficient* and *Pareto-optimal* are used interchangeably in this work.

http://dx.doi.org/10.1016/j.ins.2015.09.015
0020-0255/

solution from the huge number of Pareto-optimal ones—in other words, solving the problem of *finding Pareto-optimal points* does not necessarily mean that one has solved the practical many-objective optimization problem.

This work discusses the inclusion of preferences during the search for efficient solutions. The central idea is to allow MOEAs to bias the search procedure toward a smaller, more promising subset of the efficient set, simplifying the decision process by integrating preference information into the search process.

The main goals of this work are to investigate the effect of including preferences on the performance of evolutionary algorithms, and to present a technique that simplifies the process of biasing the multiobjective search toward the stated preferences of the DM. To verify whether the inclusion of preferences in evolutionary algorithms results in better convergence and satisfaction of the DM's interests, two evolutionary methods are compared against modified versions that take preferences into account. The inclusion of preference information into the structure of these EAs is performed by adaptively defining a region of interest (ROI) that comprises the solutions that best satisfy the preferences expressed by the DM in the form of a reference point.

This paper is organized as follows. Section 2 presents the fundamentals of multi-objective optimization, and Section 3 reviews the field of many-objective optimization. Section 4 introduces techniques for expressing preferences and including them into MOEAs. Section 5 presents the proposed method. Section 6 shows the experimental comparisons between preference and non-preference based MOEAs, as well as an experimental evaluation of the proposed method. Finally, Section 7 concludes the paper.

2. Multi-objective optimization

In multi-objective optimization problems there are m objective functions $f_i(\cdot): \mathbb{X} \mapsto \mathbb{Z}_i$, $i = 1, 2, \ldots, m$ that associate to each point $\mathbf{x} \in \mathbb{X}$ a value $f_i(\mathbf{x}) \in \mathbb{Z}_i$ that represents its *performance* value according to the ith function. $\mathbb{X}$ is referred to as the *search space*, while each $\mathbb{Z}_i$ indicates the set of values that can be output by function $f_i(\cdot)$. Together, the $\mathbb{Z}_i$ compose the *space of objectives* $\mathbb{Z}$. In this work we consider only functions that map between real spaces, *i.e.*, $\mathbb{X} = \mathbb{R}^n$, $\mathbb{Z} = \mathbb{R}^m$, and $[f_1(\cdot)\ f_2(\cdot)\ \ldots\ f_m(\cdot)]^T = \mathbf{f}(\cdot): \mathbb{R}^n \mapsto \mathbb{R}^m$.

Under these conditions, a multi-objective problem can be written without loss of generality as the problem of minimizing the objective functions:

$$\begin{aligned} &\text{minimize } \mathbf{f}(\mathbf{x}) \in \mathbb{R}^m \\ &\text{subject to } \mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n \end{aligned} \tag{1}$$

wherein $\mathcal{X}$ represents the *feasible set*, that is, the set of all possible solutions that represent acceptable solutions for the problem. It is composed of the points satisfying:

$$\begin{aligned} \mathbf{g}(\mathbf{x}) &\leq \mathbf{0} \\ \mathbf{h}(\mathbf{x}) &= \mathbf{0} \end{aligned} \tag{2}$$

in which $\mathbf{g}(\cdot): \mathbb{R}^n \mapsto \mathbb{R}^p$ and $\mathbf{h}(\cdot): \mathbb{R}^n \mapsto \mathbb{R}^q$ are, respectively, the inequality and equality constraints of the problem.[2] The image of this feasible set is called *feasible objective set*, $\mathcal{Z} = \mathbf{f}(\mathcal{X})$.

Multi-objective optimization problems are defined as (1)–(2) with $m \geq 2$. In those cases, the meaning of (1) becomes less obvious [41] since it is usually not possible to find a single alternative $\mathbf{x}^*$ that simultaneously minimizes all objectives. This problem does not arise in single-objective optimization because there is a natural ordering in scalar values [15], so the best point, if it exists, is well defined. In vector-valued functions, there is no such ordering. To enable the treatment of these cases, the concept of Pareto-dominance must be introduced.

Definition 1 (Pareto-dominance)**.** Given two alternatives $\mathbf{x}_1$, $\mathbf{x}_2 \in \mathbb{R}^n$, $\mathbf{x}_1$ Pareto-dominates $\mathbf{x}_2$ iff $f_i(\mathbf{x}_1) \leq f_i(\mathbf{x}_2)$ $\forall i \in \{1, 2, \ldots, m\}$ and $\mathbf{f}(\mathbf{x}_1) \neq \mathbf{f}(\mathbf{x}_2)$. This relationship can be expressed as $\mathbf{x}_1 \prec \mathbf{x}_2$ or $\mathbf{f}(\mathbf{x}_1) \prec \mathbf{f}(\mathbf{x}_2)$. If neither alternative dominates the other, they are said to be *incomparable* or *non-dominated*, a relation expressed by the operator $\nprec$.

Pareto-dominance induces a *partial ordering* in the space of objectives [15], meaning that some pairs of different vectors can be compared, while others are incomparable. Points that are not dominated by any other alternative within the feasible set are called *Pareto-optimal*.

Definition 2 (Pareto-optimal solution)**.** A point $\mathbf{x}^*$ is said to be *Pareto-optimal* iff $\nexists\, \mathbf{x} \in \mathcal{X}\ |\ \mathbf{x} \prec \mathbf{x}^*$, *i.e.*, if it is not dominated by any other feasible point.

Pareto-optimal solutions are also called *efficient* or *minimal* solutions in the literature [15,27], and these terms are used interchangeably throughout this text. The set of all efficient points in the space of decision variables is called the *efficient set* $\mathcal{X}^*$, and its image in the objective space is referred to as the *efficient front* $\mathcal{Z}^*$.

[2] While it is common practice in the EA literature to state the so-called box constraints ($x_{i,\text{lower}} \leq x_i \leq x_{i,\text{upper}}$, $i = 1, 2, \ldots, n$) separately in the problem formulation, it is easy to see that the bounds on the values of the decision variables x_i can be just as easily expressed using the general formulation of inequality constraints given in (2).