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Ranking and comparing evolutionary algorithms with Hellinger-TOPSIS

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ABSTRACT

When multiple algorithms are applied to multiple benchmarks as it is common in evolutionary computation, a typical issue rises, how can we rank the algorithms? It is a common practice in evolutionary computation to execute the algorithms several times and then the mean value and the standard deviation are calculated. In order to compare the algorithms performance it is very common to use statistical hypothesis tests. In this paper, we propose a novel alternative method based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to support the performance comparisons. In this case, the *alternatives* are the algorithms and the *criteria* are the benchmarks. Since the standard TOPSIS is not able to handle the stochastic nature of evolutionary algorithms, we apply the Hellinger-TOPSIS, which uses the Hellinger distance, for algorithm comparisons. Case studies are used to illustrate the method for evolutionary algorithms but the approach is general. The simulation results show the feasibility of the Hellinger-TOPSIS to find out the ranking of algorithms under evaluation.

1. Introduction

Methods for solving multicriteria decision making (MCDM) have been widely used to select a finite number of alternatives generally characterized by multiple conflicting criteria (attributes). Great efforts and significant progress have been made to the development of several MCDM approaches to solve different types of real-world problems. One of these techniques known as *technique for order preference by similarity to ideal solution* (TOPSIS) developed by Hwang and Yoon [1] evaluates the performances of the alternatives through the similarity with the ideal solution. According to this technique, the best alternative would be the one that is closest to the positive-ideal solution and farthest from the negative-ideal solution. The positive-ideal solution is the one that maximizes the benefit criteria and minimizes the cost criteria. The negative-ideal solution maximizes the cost criteria and minimizes the benefit criteria.

TOPSIS is widely used to treat real-world decision making problems and it has been generalized to deal with a variety of information types. For example, there are many adaptations of TOPSIS to deal with interval numbers [2,3], probability distributions [4], fuzzy information [5–9] intuitionistic fuzzy information [10], interval-valued intuitionistic fuzzy information [11,12], among many others. For a broad survey about the TOPSIS the interested reader shall refer to [13].

A great difficulty in evolutionary computation is the comparison of algorithms. Usually, the algorithms are applied several times to multiple benchmarks. Then, the results are analyzed by means of statistical hypothesis tests [14,15]. The statistical tests can detect if there are differences between the performances of the algorithms. The problem is if there are differences, which algorithm is the best one? To use statistical tests in this step, it is necessary to make pairwise comparisons between the algorithms. Obviously, the number of tests required increases greatly with the number of algorithms being analyzed. This is problematic first because the tiresome work of comparing each pair of algorithms; secondly and more importantly, the probability of making an error increases. When multiple hypothesis tests are being performed, one can only choose between increasing the probability of Type 1 Error or losing test power [22].

Recently, the TOPSIS method has also been extended to treat, in a direct way, data expressed as probability distributions by means of the Hellinger distance [16]. It means that the TOPSIS with Hellinger distance have opened a new possibility for ranking alternatives expressed in terms of probability distributions in the context of MCDM problems.

Due to the stochastic nature of the evolutionary algorithms, in many cases the performance of the algorithms are expressed in

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terms of mean and standard deviation. However, since these quantities are usually random variables, a direct comparison of these values may not be as meaningful as desired. For this reason, the usual approach is to perform statistical hypothesis tests to compare the algorithms.

We present a new approach to support the selection of the best algorithms by applying the Hellinger-TOPSIS and reducing the quantity of hypothesis tests to be performed. In addition, in most situations it is not necessary to know the exact distribution of the solution of an algorithm. By the Central Limit Theorem (CLT), we know that $\bar{X} \xrightarrow{D} N(\mu, \sigma^2/r)$. Therefore, if the algorithm is performed r times with r sufficiently large, we can approximate the distribution of the sample mean by the Gaussian distribution. Then, the Hellinger-TOPSIS can be easily applied to provide a rank order of the algorithms in a very easy and direct way, using the Hellinger distance between two Gaussians distributions. In the context of algorithms comparison, the alternatives consist of multiple algorithms and the criteria are the benchmarks. Assuming the validity of CLT, the method is simplified and it gets easier to be applied.

The aim of this work is to present a tool to aid in selecting the best algorithms when applied to multiple benchmarks. The rest of this article is organized as follows: Section 2 describes the TOPSIS. In Section 3, we present the Hellinger-TOPSIS to deal with decision matrix with ratings expressed in terms of probability distributions. In Section 4, two case studies are analyzed, where different versions of genetic algorithms are applied to a suite of benchmarks in order to illustrate the feasibility of the approach. In Section 5, conclusions and directions for future work are given.

2. Technique for order preference by similarity to ideal solution – TOPSIS

Let us consider the decision matrix D, which consists of a*lternatives* and *criteria*, described by

$$D = \begin{array}{c} \\ A_1 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \dots & C_n \end{array} \\ \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are viable alternatives, and $C_1, C_2, \ldots, C_n$ are criteria, x_{ij} indicates the rating of the alternative A_i according to criteria C_j. The weight vector W is composed of the individual weights $W = (w_1, w_2 \ldots, w_n)$ with $w_j (j = 1, \ldots, n)$ for each criterion C_j satisfying $\sum_{j=1}^{n} w_j = 1$. In general, the criteria are classified into two types: *benefit* and *cost*. The *benefit* criterion means that a higher value is better while for *cost* criterion is valid the opposite. The data of the decision matrix D come from different sources so, in general, it is necessary to normalize them in order to obtain a dimensionless matrix. The normalized value r_{ij} can be calculated as

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \text{ with } i = 1, \ldots, m;\ j = 1, \ldots, n \tag{1}$$

or

$$r_{ij} = \frac{x_{ij}}{\max x_{ij}}, \text{ with } i = 1, \ldots, m;\ j = 1, \ldots, n \tag{2}$$

The normalized decision matrix $R = [r_{ij}]_{mxn}$ represents the relative rating of the alternatives. After normalization, we calculate the weighted normalized decision matrix $P = [p_{ij}]_{mxn}$ with $i = 1, \ldots, m$, and $j = 1, \ldots, n$ by multiplying the normalized decision matrix by its associated weights. The weighted normalized value p_{ij} is calculated as:

$$p_{ij} = w_j r_{ij} \text{ with } i = 1, \ldots, m, \text{ and } j = 1, \ldots, n. \tag{3}$$

The TOPSIS is described in the following steps [17]:

Step 1: Identify the positive ideal solutions (PIS) A^+ (benefits) and the negative ideal solutions (PIS) A^- (costs) as follows:

$$A^+ = (p_1^+, p_2^+, \ldots, p_m^+) \text{ where } p_j^+ = (\max_i\ p_{ij}, j \in J_1; \min_i\ p_{ij}, j \in J_2) \tag{4}$$

$$A^- = (p_1^-, p_2^-, \ldots, p_m^-) \text{ where } p_j^- = (\min_i\ p_{ij}, j \in J_1; \max_i\ p_{ij}, j \in J_2) \tag{5}$$

where J_1 and J_2 represent *benefit* and *cost* criteria, respectively.

Step 2: Calculate the separation measures from the positive ideal solutions A^+ (benefits) and the negative ideal solutions A^- (costs) for each alternative A_i, respectively as follows:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (p_j^+ - p_{ij})^2} \quad \text{with } i = 1, \ldots, m. \tag{6}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (p_j^- - p_{ij})^2} \quad \text{with } i = 1, \ldots, m. \tag{7}$$

Step 3: Calculate the relative closeness coefficient ξ_i for each alternative A_i with respect to positive ideal solution as given by

$$\xi_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{8}$$

Step 4: Rank the alternatives according to the relative closeness coefficients. The best alternatives are those that have higher value ξ_i and therefore should be chosen.

The extension of TOPSIS to Hellinger-TOPSIS is described in the following.

3. The Hellinger-TOPSIS

The decision matrix D consisting of a*lternatives* and *criteria* is described by

$$D = \begin{array}{c} \\ A_1 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \dots & C_n \end{array} \\ \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are alternatives, $C_1, C_2, \ldots, C_n$ are criteria, x_{ij} indicates the rating of the alternative A_i with respect to criterion C_j described in terms of probability distributions. In the context of algorithms comparison, the alternatives consists of the algorithms and the criteria are the benchmark problems.

In order to define a method that deals directly with information in the form of probability distributions, we need to answer two questions: given two probability distributions which one is higher/preferable? and how far are they from each other? Next, these issues have been addressed in [16] and here we provide some necessary definitions.

Definition 1 *([18])*. Let f and g be two probability density functions (pdf). The Hellinger distance between f and g is given by

$$D_H(f, g) = \sqrt{\frac{1}{2} \int_{\mathbb{R}} \left(\sqrt{f(x)} - \sqrt{g(x)}\right)^2 dx} \tag{9}$$