Knowledge-Based Systems 85 (2015) 131–142

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A new uniform evolutionary algorithm based on decomposition and CDAS for many-objective optimization

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ARTICLE INFO

Article history:
Received 21 December 2014
Received in revised form 14 March 2015
Accepted 21 April 2015
Available online 5 May 2015

Keywords:
Multi-objective optimization
Decomposition
Uniform design
Weight vector
Many-objective optimization problems

ABSTRACT

The convergence and the diversity are two main goals of an evolutionary algorithm for many-objective optimization problems. However, achieving these two goals simultaneously is the difficult and challenging work for multi-objective evolutionary algorithms. A uniform evolutionary algorithm based on decomposition and the control of dominance area of solutions (CDAS) is proposed to achieve these two goals. Firstly, a uniform design method is utilized to generate the weight vectors whose distribution is uniform over the design space, then the initial population is classified into some sub-populations by these weight vectors. Secondly, an update strategy based on decomposition is proposed to maintain the diversity of obtained solutions. Thirdly, to improve the convergence, a crossover operator based on the uniform design method is constructed to enhance the search capacity and the CDAS is used to sort solutions of each sub-population to guide the search process to converge the Pareto optimal solutions. Moreover, the proposed algorithm compare with some efficient state-of-the-art algorithms, e.g., NSGAII-CDAS, MOEA/D, UMOEA/D and HypE, on six benchmark functions with 5–25 objectives are made, and the results indicate that the proposed algorithm is able to obtain solutions with better convergence and diversity.

1. Introduction

Since there are many problems with several optimization objectives or criteria in the real world [1], multi-objective optimization has become a hot research topic. Unlike single-objective optimization problem, multi-objective optimization problem has a series of non-inferior alternative solutions, also known as Pareto optimal solutions (the set of Pareto optimal solutions in objective space is called Pareto front [2]). Multi-objective evolutionary algorithms (MOEAs) is an effective method for solving multi-objective problems because they can handle a set of solutions in parallel, so MOEAs can obtain multiple Pareto optimal solutions in a single run. Almost all well-known and frequently-used MOEAs, which have been proposed in the last twenty years [3–7], are based on Pareto dominance. Such Pareto dominance-based algorithm usually deals well with two or three objectives problems, but their searching and selecting abilities are often severely degraded by the increased number of objectives [8]. This is explained by the fact that, the number of solutions is constant, then the size of non-dominance area of solutions will increase with the increase of the number of objectives, which will make the Pareto dominance-based fitness evaluation generate very weak selection pressure toward the Pareto front (PF). Moreover, when the number of objectives increases, the dimensions of PF and proportion of non-dominated solutions in the population are also increasing, which can lead to difficult to obtain a set of solutions which have good diversity and converge to the PF for a MOEA. How to effectively solve many-objective optimization problems has caused the attention of many scholars and has become the hot topic.

Currently, evolutionary algorithms for dealing with many-objective problems can be divided into three categories. The first category uses an indicator function, such as the hypervolume [9–11], as the fitness function. This kind of algorithms is also referred to as IBEAs (indicator-based evolutionary algorithms), and their high search ability has been shown in the literature [12]. Bader and Zitzler [13] propose a fast hypervolume-based many-objective optimization algorithm (HypE) which uses Monte Carlo simulation to speed up approximately the exact hypervolume values. However, one of their main drawbacks is the computation time for the hypervolume calculation which exponentially

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http://dx.doi.org/10.1016/j.knosys.2015.04.025
0950-7051/

increases with the number of objectives [14], and even if the hypervolume values are calculated by Monte Carlo approximations, its running time is more than 10 h after 50,000 objective function evaluations for seven-objective problems [15]. This limits the application of hypervolume indicator-based evolutionary algorithms to many-objective optimization problems.

The second category takes advantages of solution ranking methods. Specifically, solution ranking methods are used to discriminate among solutions in order to enhance the selection pressure toward the PF, which makes sure the solutions are able to converge to the PF. At present, numerous approaches have been proposed for ranking solutions when dealing with many-objective problems. Bentley and Wakefield [16] proposed ranking composition methods which extract the separated fitness of every solution into a list of fitness values for each objective. Kokolo and Hajime [17] proposed a relaxed form of dominance (RFD) to deal with what they called dominant resistant solutions, i.e., these solutions that are extremely inferior to others in at least one objective, but hardly-dominated. Dai and Wang [18] proposed a contraction method on non-dominance area to rank solutions to improve the convergence. Sato et al. [19] proposed a method to strengthen or weaken the selection process by expanding or contracting the solutions' dominance area which is the CDAS. When the dominance of solutions is contracted, the CDAS can guide the search process to converge to the true PF, and the CDAS has been used in some MOEAs to improve the convergence (e.g. [20–22]). However, when the CDAS improves the convergence, it is hard to maintain the diversity.

The third category utilizes the scalarizing functions to deal with the many-objective problems. According to the literatures [23–25], scalarizing function-based algorithms could better deal with many-objective problems than Pareto dominance-based algorithms. The main advantage of scalarizing function-based algorithms is the simplicity of their fitness evaluation which can be easily calculated. The representative MOEA in this category is MOEA/D [26] (multi-objective evolutionary algorithm based on decomposition), which works well on a wide range of multi-objective problems with many objectives, discrete decision variables and complicated Pareto sets [27–29]. However, for many-objective problems, the decomposition-based evolutionary algorithms will face the following two problems: (1) how to generate a set of uniformly distributed weight vectors to uniformly divide the objective space; (2) how to make each sub-problem converge to the Pareto optimal solutions. In MOEA/D [26], the uniformity of the used weighted vectors determines the uniformity of the obtained non-dominated optimal solutions, however, the used weighted vectors in MOEA/D will be not very uniform for some problems and the size N of these weighted vectors should satisfy the restriction $N = C_{H+m-1}^{m-1}$. Thus N cannot be freely assigned and it will increase nonlinearly with m, where m is the number of objectives and H is an integer. This restricts the application of MOEA/D to a certain extent in multi-objective optimization problems. Therefore, for many-objective problems, how to set weight vectors is a very difficult but critical task, and it is necessary to consider an efficient and simple method to produce the weight vectors [30]. Tan and Ma [29,31] use the uniform design to yield weight vectors and design a uniform design multi-objective evolutionary algorithm based on decomposition for many-objective optimization problems, but the algorithm only tests five-objective problems. In addition, the angles between the weight vectors will increase with the number of objectives (too many weight vector can increase the amount of calculation), which can lead to weakening the correlation between the sub-problems and may cause the convergence performance of decomposition-based algorithms to degrade.

In this paper, a new evolutionary algorithm based on decomposition (UCEA/D) is intended to find a set of solutions which have good diversity and converge to the true PF for many-objective optimization problems. Firstly, we adopt a uniform design method to generate a set of points which are uniformly distributed on a unit sphere, and the points are the weight vectors. The number of weight vectors only needs to be larger than the number of objectives (in general, the number of the weight vectors is greater than the number of objectives). Then the objective space is divided into a set of sub-objective spaces by the set of weighted vectors. Secondly, a sup-population strategy is used to enhance the local search ability of the proposed algorithm. Each sub-problem generated by decomposition will have a sub-population and use the information provided by its sub-population to improve the convergence performance. Thirdly, The CDAS is used to rank the solutions of each sub-population, which will provide stronger selection pressure toward the PF than Pareto dominance. Fourthly, an update strategy based on decomposition is proposed to maintain the diversity. Finally, a crossover operator is constructed by a uniform method to improve the search capacity. Moreover, the experiments demonstrate that UCEA/D can significantly outperform UMOEA/D, MOEA/D, NSGAII-CDAS (NSGAII based on CDAS) and HypE on a set of test instances.

The rest of this paper is organized as follows: Section 2 introduces the main concepts of the multi-objective optimization, CDAS and uniform design; Section 3 presents a new multi-objective evolutionary algorithm; while Section 4 shows the experiment results of the proposed algorithm and the related analysis; finally, Section 5 draws the conclusions and proposes the future work.

2. Related knowledge

In this section, the main concepts of the multi-objective optimization, CDAS and uniform design are introduced.

2.1. Multi-objective optimization

A multi-objective optimization problem can be formulated as follows [33]:

$$\begin{cases} \min\ F(x) = (f_1(x), f_2(x), \ldots, f_m(x)) \\ \text{s.t. } g_i(x) \leqslant 0, \quad i = 1, 2, \ldots, e_1 \\ h_j(x) = 0, \quad j = 1, 2, \ldots, e_2 \end{cases} \tag{1}$$

where $x = (x_1, \ldots, x_n) \in X \subset R^n$ is called a decision vector and X is n-dimensional decision space. $f_i(x)(i = 1, \ldots, m)$ is the ith objective to be minimized, $g_i(x)(i = 1, 2, \ldots, e_1)$ defines ith inequality constraint and $h_j(x)(j = 1, 2, \ldots, e_2)$ defines jth equality constraint. Furthermore, all the constraints determine the set of feasible solutions which are denoted by Ω. To be specific, we try to find a feasible solution $x \in \Omega$ minimizing each objective function $f_i(x)(i = 1, \ldots, m)$ in F. In the following, four important definitions [32] for multi-objective problems are given.

Definition 1 (*Pareto dominance*). Pareto dominance between solutions $x, z \in \Omega$ is defined as follow. If

$$\begin{aligned} &\forall i \in \{1, 2, \ldots, m\} f_i(x) \leqslant f_i(z) \\ &\wedge \exists i \in \{1, 2, \ldots, m\} f_i(x) < f_i(z) \end{aligned} \tag{2}$$

are satisfied, x dominates (Pareto dominate) z (denoted $x \succ z$).

Definition 2 (*Pareto optimal*). A solution vector x is said to be Pareto optimal with respect to Ω, if $\nexists z \in \Omega : z \succ x$.