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An improved robust topology optimization approach using multiobjective evolutionary algorithms

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ABSTRACT

Robust topology optimization has gained importance during the last years. This paper presents a robust approach to topology optimization using multiobjective evolutionary algorithms. A key contribution of our approach is that our optimization model handles structural robustness through the first two objectives, namely, the expected compliance and its variance; whereas a third objective incorporates the volume of the structure and tackles the sizing optimization problem. Finally, a major contribution of the proposed approach is that it returns a Pareto frontier showing the designer an array of possible solutions and unveiling the existing tradeoff between the different problem objectives, namely the expected compliance, variance of compliance, and volume of the structure.

1. Introduction

Since the seminal work by Taguchi [1], robust design has gained ever-increasing importance among researchers and practitioners [2]. The goal of robust design is to obtain solutions that perform well under uncertain conditions [3]. According to Taguchi, when dealing with topology optimization there are two types of parameters that should be considered: (1) the *control parameters*, which are to be optimized to achieve the problem objectives, and (2) the *noise factors*, which affect the accuracy of the result; these are relatively out of the designer's control (e.g., environmental conditions). Designing a product for the expected conditions (i.e., deterministic approach) may not be optimal from the operational point of view, as the variation of the noise factors may deem the design impractical or simply infeasible. Therefore, the central point in robust design is to handle the uncertainty arising from the noise factors, which ultimately plays a significant role within the structural designer's decision-making process.

In classic structural topology optimization, one of the main objectives is to obtain the optimal material distribution at a given structural design point under known (i.e., deterministic) material, geometrical, and structural performance conditions [4]. In this relatively new field of research, some of the most popular methodologies include, among others, the homogenization method, developed by Bendsøe and Kikuchi [5], the solid isotropic material with penalization method (SIMP) [6,7], level set methods [8,9], evolutionary structural optimization (ESO) [10], stochastic search methods such as genetic algorithms [11–14], simulated annealing [15,16], and hybrid methods such as simulated annealing-SIMP (SA-SIMP) [17]. Recently, Rao [19] highlighted the need for a multiobjective approach to structural optimization. He also introduced a game theory approach for finding a Pareto frontier of optimal structural solutions [20] and developed an approach based on genetic algorithms for multiobjective optimization of structures [21]. For a thorough treatment of structural optimization, the reader is directed to Rao [22] and Cheng and Li [23] for an overview of constraint handling approaches. An excellent review of the state-of-the-art in topology optimization was published by Rozvany [18].

Recently, there have been important efforts to include both parameter and model uncertainties into the optimization problem. Developments in reliability-based topology optimization (RBTO) include applications in many areas including civil infrastructure, electromagnetics, heat transfer, and coupled systems [25]. In the field of structural topology optimization, the pioneering work which included variability in the analysis was published by Kharmanda et al. [24]. They modified the standard deterministic approach to allow for some uncertainty to be added into the input parameters; thus leading to a solution with a given reliability level. Some other RBTO strategies have been developed to take into account the uncertainties associated to various aspects of the

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problem such as the material properties, loading, and boundary conditions [26,27]. In addition, among the optimization strategies, there have been also numerous developments, which include RBTO models based on evolutionary structural optimization algorithms (ESO) [28]; models that use probability and convex set hybrid models [29]; models based on level set methods [30]; models that focus on the topological solution by including uncertainties in load location and magnitude and structural nodal locations [31]; models that include non-probabilistic uncertainties [32–34]; and single-loop approaches [35]. Schuëller and Jensen [36] provide an excellent overview of computational methods for stochastic structural optimization, some of which are of great use to find optimal topological solutions. Also, Deb et al. [37] and Ye et al. [38] present up to date surveys on evolutionary algorithms for reliability-based optimization problems.

The concept of robustness in structural optimization has opened an entire new field of research that has gained momentum during the last few years. One of the pioneering papers on structural robust topology optimization is due to Seepersad et al. [39] who optimized cell structures taking into account uncertainties in the boundary conditions. Also, Yildiz et al. [40] introduced a hybrid multiobjective shape design optimization method which used Taguchi's method to achieve robustness. Kogiso et al. [41] presented a sensitivity based approach which optimized compliance mechanisms. Finally, Conti et al. [42] and Chen et al. [43] combined level set methods and robust design formulations to find optimal solutions. Other approaches include robust truss topology optimization [44,45] and fuzzy-based robust structural optimization [46].

None of the approaches mentioned above has tackled the structural robust topology optimization problem from a multiobjective *a-posteriori* perspective. Thus, in this paper we develop a robust topology optimization approach, which uses multiobjective evolutionary algorithms (MOEAs) to obtain structural topologies such that the displacement and compliance are minimal with respect to uncertain input variables like force and material properties. Also, we propose a novel approach to sizing optimization by directly incorporating this objective into the problem.

This paper is organized as follows. In Section 2, we present a brief description of the proposed robust multiobjective evolutionary algorithm. Section 3 presents the computational results for different experiments taking into account various sources of uncertainty and comparing our robust approach against deterministic approaches. Section 4 presents the results and a critical discussion about the model. Finally, we outline some concluding remarks.

2. Structural topology optimization problem formulation

A classical approach to topology optimization seeks for an ideal structure $\mathbf{x}^*$ with minimal compliance $c(\mathbf{x}^*)$; this is equivalent to maximizing the stiffness subject to a volume constraint on the structure. Hence, the robust counterpart can be stated as the following bi-objective optimization problem:

$$\min \quad f_1(\mathbf{x},\boldsymbol{\omega}) = E(c(\mathbf{x},\boldsymbol{\omega})) \tag{1}$$

$$\min \quad f_2(\mathbf{x},\boldsymbol{\omega}) = Var(c(\mathbf{x},\boldsymbol{\omega})) \tag{2}$$

$$\text{subject to,} \quad \mathbf{F} = \mathbf{KU} \tag{3}$$

$$V(\mathbf{x}) \leqslant V_f \tag{4}$$

$$\mathbf{x} \geqslant \mathbf{0} \tag{5}$$

where (1) and (2) minimize the expected value $E(c(\mathbf{x},\boldsymbol{\omega}))$ and the variance $Var(c(\mathbf{x},\boldsymbol{\omega}))$ of the response $c(\mathbf{x},\boldsymbol{\omega})$, subject to different sources of uncertainty, $\boldsymbol{\omega}$; for example, the load magnitude and angle of application. Eq. (3) states the mechanical equilibrium conditions, where $\mathbf{F}$ is the force vector, $\mathbf{K}$ the stiffness matrix, and $\mathbf{U}$ the displacement vector. The constraint (4) establishes that the volume of the structure $V(\mathbf{x})$ should not be larger than the volume fraction, V_f. Finally, (5) establishes the non-negativity of the decision variables.

The optimization problem defined by (1)–(5) can be rewritten by treating the volume of the structure not as a constraint but rather as an additional problem objective. Note that this constraint-handling technique has been used with success in lieu of the more classical penalty function approach often used in single-objective optimization. By adding the constraint as an additional objective it is possible to avoid the always difficult parameter (penalty) tuning [53,55]. By adding this third objective, the sizing optimization decisions are included into the problem formulation, giving the designer more information about the trade-off between volume and stiffness. The sizing and robust topology optimization problem can be stated as follows:

$$\text{Min} \quad f_1(\mathbf{x},\boldsymbol{\omega}) = E(c(\mathbf{x},\boldsymbol{\omega})) \tag{6}$$

$$\text{Min} \quad f_2(\mathbf{x},\boldsymbol{\omega}) = Var(c(\mathbf{x},\boldsymbol{\omega})) \tag{7}$$

$$\text{Min} \quad f_3(\mathbf{x},\boldsymbol{\omega}) = V(\mathbf{x}) \tag{8}$$

$$\text{subject to} \quad \mathbf{F} = \mathbf{KU} \tag{9}$$

$$\mathbf{x} \geqslant \mathbf{0} \tag{10}$$

3. Robust topology optimization using MOEAs

3.1. Robust design-based decisions

Robust design optimization consists of finding structural solutions such that the variability of the response – i.e., performance measure – is relatively insensitive to the uncertainty of the material properties and the demands. Most approaches to structural robustness evaluate the structural response in terms of displacement; however, others use the first and second moments of the response function. For an overview of the robust optimization conceptual and theoretical aspects see Beyer and Sendhoff [3]. Research in robust design was fostered by the pioneering work of Taguchi [1], who devised an optimization methodology based on maximizing the *signal to noise ratios* (SNR), which evaluates the *mean response variation* of the designs with respect to the *noise factors*. However, for large scale problems, the Taguchi approach faces efficiency shortcomings, since it is based on a careful design of experiments.

Based on the preference elicitation process of the decision maker, there are two mainstream alternatives to find robust solutions under multiple criteria. An *a-priori* approach combines the objectives (i.e., response measure and its variability) into a single objective using a weighted sum or aggregating function [43]. On the other hand, the *a-posteriori* approach unveils the (approximate) Pareto front by simultaneously minimizing the conflicting objectives [47]. This paper focuses on the latter approach.

3.2. Multiobjective evolutionary algorithms

Evolutionary algorithms (EAs) have become one of the leading methods for tackling multiobjective optimization problems using *a-posteriori* articulation of preferences. EAs are bio-inspired stochastic search procedures in which an initial set of solutions (population) evolve using selection, recombination, and perturbation mechanisms (such as local search and mutation) [48].