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Mahalanobis distance based on fuzzy clustering algorithm for image segmentation

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ABSTRACT

Conventional Fuzzy C-means (FCM) algorithm uses Euclidean distance to describe the dissimilarity between data and cluster prototypes. Since the Euclidean distance based dissimilarity measure only characterizes the mean information of a cluster, it is sensitive to noise and cluster divergence. In this paper, we propose a novel fuzzy clustering algorithm for image segmentation, in which the Mahalanobis distance is utilized to define the dissimilarity measure. We add a new regularization term to the objective function of the proposed algorithm, reflecting the covariance of the cluster. We experimentally demonstrate the effectiveness of the proposed algorithm on a generated 2D dataset and a subset of Berkeley benchmark images.

1. Introduction

Until now, a great deal of attention has been paid to image segmentation [1–3]. There are many ways to implement the segmentation, and they can be divided into three categories in terms of their mathematic foundations: threshold-based [4,5], cluster-based [6,7] and statistics-based algorithms [8,9]. Among of them, cluster-based algorithms are widely used in image segmentation where Fuzzy C-means (FCM) algorithm [10–12] is one of the most popular methods due to its simplicity and extendibility. FCM algorithm uses the Euclidean distance to describe the dissimilarity measure between pixels and cluster centers. Besides the algorithm introduces the fuzzy set theory to define a fuzzy membership function and the exponentially weight of the fuzzy membership function can express the fuzziness of the objective function in the FCM algorithm. It means that if the pixel has a bigger dissimilarity measure with a cluster center it has a less degree to belong to the cluster and vice versa [13]. From image segmentation point of view, the FCM algorithm does not take the relations among neighboring pixels into account. Apart from that, the exponential weights in its objective function are a rather unnatural choice [14,15] and the divergences of intensities of pixels in segmented regions are also ignored. These make FCM algorithm sensitive to noise and cluster divergence.

To improve the FCM algorithm, Kim et al. [16] suggested a fuzzy model which can express a given unknown system with a few fuzzy rules. Krishnapuram and Keller [17] proposed the Possibilistic C-Means (PCM) algorithm, in which the membership of the data-points is interpreted as its possibility belonging to a class and an appropriate objective function is constructed from a possibilistic point of view. Detroja et al. [18] proposed a new approach for fault detection and isolation based on the possibilistic clustering algorithm. Pal et al. [19] proposed Possibilistic-Fuzzy C-Means (PFCM) model and used it as a candidate for fuzzy rule-based system identification. Gong et al. [20] introduced a tradeoff between weighted fuzzy factor and a kernel metric to improve the FCM algorithm. Tan et al. [21] presented a novel initialization scheme to determine the cluster number and obtain the initial cluster centers for the FCM algorithm. In spite of those ways, the FCM algorithm improves mainly the following three aspects: neighborhood system, objective function and dissimilarity measure.

From a neighborhood relationship point of view, Ahmed et al. [22] presented FCM with constrains (FCM_S) algorithm, in which the Euclidean distance from a pixel to a cluster center and the Euclidean distance from its neighboring pixels to the cluster center are calculated, respectively, and then a dissimilarity measure between the pixel and the cluster center is defined by using the weighted averaged value of those distances. Although they introduced neighborhood information into the objective function, the algorithm is insufficient to outliers. Moreover, in algorithm iteration, the Euclidean distances from neighboring pixels to cluster centers needed to be calculated repeatedly. This makes the algorithm time-consuming. Wherefore, Szilagyi et al. [23] proposed the Enhanced FCM (EnFCM) algorithm in such a way that a linearly-weighted sum image is in advance formed by weightily averaging each pixel and its neighborhoods. Although the smoothed image

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may decrease the affect of noise, the detail information is lost during the smoothing process and this will lead to misclassifications. In addition, the weighted coefficient which may affect the segmentation result is given by users. Cai et al. [24] defined a similarity measure which is related to neighborhood and spatial information as the weighting coefficient in Fast Generalized FCM (FGFCM). They used two parameters to control the influence of neighborhood system and spatial information in regularization. To speed up the algorithm, EnFCM and FGFCM carry out image segmentation on gray levels rather than pixels. Unlike [24], Krinidis and Chatzis [25] proposed the Fuzzy Local Information C-Mean (FLICM) algorithm and used spatial and spectral information of neighboring pixels to define a fuzzy factor, which controls the balance between noise reduction and detail protection. The algorithm is performed on the original image rather than filtered image as used in [23,24] which may cause detail missing.

In addition, some researchers attempted to redefine the objective functions to improve FCM algorithm. In the proposed Entropy-based FCM (EFCM), Miyamoto and Mukaidono [26] used a regularization, which is implemented by information entropy, to define their objective function and a coefficient of the regularization to indicate the fuzziness of the objective function. However, EFCM may lead to misclassification for divergent clusters (clusters with large variances). To overcome this problem Ichihashi et al. [27] proposed KL (Kullback–Leibler) information-based FCM (KLFCM), in which an additional variable in regularization is used to improve the KL information. Furthermore, Miyamoto et al. [28] proved that the additional variable can control the cluster size.

Apart from improving the objective function, the definitions of dissimilarity measures can also improve the FCM algorithm. Euclidean distance defines the straight line distance between two data points in feature space. As a dissimilarity measure, it is sensitive to noise because it does not consider covariance of data points. Therefore, Euclidean distance based FCM algorithm can be improved by defining dissimilarity measures with other distance metrics. Chen and Zhang [29] used the Kernel-induced distance as dissimilarity measure in FCM algorithm and showed that the kernel method is an effective approach to construct a robust image clustering algorithm. However, the Kernel-induced distance utilizes only mean information of clusters in an image. To take more information into consideration, Carvalho et al. [30] used the mean vector of an image and a symmetric matrix to define the adaptive quadratic distance as a dissimilarity measure, where the symmetric matrix can be defined in several ways, hence different definitions lead to different results. Han et al. [31] applied the divergence distance which combines both mean and covariance information to the FCM algorithm. By comparing the results of algorithms with divergence distance and Euclidean distance, they proved the effectiveness of the algorithm based on divergence distance. Mahalanobis distance [32] is also defined by mean and covariance of a cluster but Krishnapuran and Kim [33] proved that the distance cannot be used directly as dissimilarity measure for designing cluster algorithm. So Liu et al. [34] used Mahalanobis distance by defining the matrix through pixels and their mean. When the matrix is a unit matrix, the Mahalanobis distance is equal to the Euclidean distance [35]. Liu et al. [34,36] added the log determinant of the covariance matrix to Mahalanobis distance to define a new dissimilarity measure. A data point that falls together with the mean of a cluster would have a non-zero dissimilarity measure indicating that this measure is not a well defined distance. This paper proposed a Mahalanobis distance based FCM (MFCM) algorithm in which Mahalanobis distance is used to define the dissimilarity measure and a regularization term is added to the objective function which is defined in reference [27].

The paper is organized as follows. In Section 2, the differences of Euclidean and Mahalanobis distance are compared and the proposed algorithm is described in detail. In Section 3, a generated dataset on two dimension (2D) plane, a synthetic image and a subset of Berkeley benchmark images are segmented and the effectiveness of the proposed algorithm is evaluated qualitatively and quantitatively. A conclusion is given in Section 4.

2. Fuzzy clustering algorithm based on Mahalanobis distance

Let $\boldsymbol{X} = \{\boldsymbol{x}_i : i = 1, 2, \ldots, N\}$ be the observed image, where i is in responding of the pixel index, $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{id})^T$ is the feature vector of pixel i, d is the dimension of the pixel, and N is the number of pixels in image $\boldsymbol{X}$.

The objective function of the FCM algorithm can be formulated as

$$J_{FCM} = \sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij}^m d_{ij} \tag{1}$$

where c is the number of clusters, and j is the index of the cluster, $\boldsymbol{U} = [u_{ij}]_{N\times c}$ is the membership matrix expressing the fuzzy segmentation, u_{ij} represents the degree of membership with which $\boldsymbol{x}_i$ belongs to the jth cluster and satisfies $\sum_{j=1}^{c} u_{ij} = 1$. The fuzzy factor m is the weighting exponent of u_{ij} and describes the degree of fuzziness of the algorithm, $d_{ij} = \|\boldsymbol{x}_i - \boldsymbol{\mu}_j\|_2$ represents the Euclidean distance measuring the dissimilarity between the pixel vector $\boldsymbol{x}_i$ and the mean vector of the jth cluster $\boldsymbol{\mu}_j = (\mu_{j1}, \mu_{j2}, \ldots, \mu_{jd})^T$. The FCM algorithm based on the Euclidean distance is sensitive to noise and the weighting exponent in objective function is suboptimal [37].

Miyamoto and Mukaidono [26] proposed to use an entropy term as regularization for the objective function

$$J_{EFCM} = \sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij} d_{ij} + \lambda \sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij}\log u_{ij} \tag{2}$$

where λ is the degree of the fuzziness of the algorithm. However, this objective function is sensitive to divagating clusters. Ichihashi et al. [27] proposed the KLFCM algorithm which improved the EFCM algorithm by using the KL information instead of entropy, that is,

$$J_{KLFCM} = \sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij} d_{ij} + \lambda \sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij}\log\left(\frac{u_{ij}}{\alpha_j}\right) \tag{3}$$

where the elements of $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \ldots, \alpha_c\}$ control the sizes of clusters and avoid misclassification of edge points belonging to the larger cluster being segmented to the adjacent smaller cluster [28].

Using Eq. (3) as objective function together with the Euclidean distance as dissimilarity measure still shows being highly sensitive to noise and divergence of clusters. To overcome this shortcoming, we introduce to use the Mahalanobis distance

$$d_{ij} = (\boldsymbol{x}_i - \boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_j^{-1} (\boldsymbol{x}_i - \boldsymbol{\mu}_j) \tag{4}$$

where $\boldsymbol{\Sigma}_j$ is the covariance matrix of the jth cluster. When $\boldsymbol{\Sigma}_j$ is a unit metric, the Mahalanobis distance reduces to the Euclidean distance. To compare the Euclidean distance with the Mahalanobis distance, a dataset with three clusters has been generated in 2D plane where the data points subject to a Gaussian distribution. The mean vectors, covariance matrices and numbers of data points in clusters are listed in Table 1 where μ_{jk} ($j = 1, 2, 3; k = 1, 2$) represents the components of the mean vector of the jth cluster and σ_{jk} ($j = 1, 2, 3; k = 1, 2, 3, 4$) represents the components of covariance of the jth cluster. Fig. 1 shows the generated dataset, in which the data points for Clusters 1, 2 and 3 are in red, green and blue, respectively, and the black and pink lines indicate the Euclidean distances (E_d) and Mahalanobis distances (M_{dr}, M_{dg}, M_{db}). It is clear