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Fast algorithms for least-squares-based minimum variance spectral estimation

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ABSTRACT

The minimum variance (MV) spectral estimator is a robust high-resolution frequency-domain analysis tool for short data records. The traditional formulation of the minimum variance spectral estimation (MVSE) depends on the inverse of a Toeplitz autocorrelation matrix, for which a fast computational algorithm exists that exploits this structure. This paper extends the MVSE approach to two data-only formulations linked to the covariance and modified covariance cases of least-squares linear prediction (LP), which require inversion of *near-to-Toeplitz* data product matrices. We show here that the *near-to-Toeplitz* matrix inverses in the two new fast algorithms have special representations as sums of products of triangular Toeplitz matrices composed of the LP parameters of the least-squares-based formulations. Fast algorithm solutions of the LP parameters have been published by one of the authors. From these, we develop fast solutions of two least-squares-based minimum variance spectral estimators (LS-based MVSEs). These new MVSEs provide additional resolution improvement over the traditional autocorrelation-based MVSE.

1. Introduction

High-resolution spectral estimation is used to provide enhanced spectral feature detailing in many practical applications, such as acoustics, communication, radar, biomedicine, economics, and many other fields. The minimum variance spectral estimation (MVSE) was originally introduced by Capon [1] for use in multi-dimensional seismic array frequency-wavenumber analysis. Lacoss [2] reformulated Capon's MVSE for application to one-dimensional (1-D) time-series analysis. Some recent work has been done to improve the characteristics and applications of the MVSE. For example, Krolik and Eizenman [3] facilitated MVSE for broad-band source location. Lee and Munson [4] reformulated the spatially variant apodization (SVA) as a special version of the MVSE to implement image reconstruction from partial Fourier data. Frazho and Sherman [5] discussed the convergence of the MVSE in a non-stationary noise environment.

The primary disadvantage of the MVSE, as defined later in Eq. (5), over the more conventional fast Fourier transform (FFT)-based periodogram is the computational burden if it must be directly computed. First, the evaluation of the MVSE involves the calculation of an inverse autocorrelation matrix, which is computationally intensive and uses a large amount of memory to store the matrix, especially if the order is large. Second, the MVSE has to be evaluated over all frequencies of interest. Typically, this second operation can be even more computationally intensive if the number of frequencies is high. These computational disadvantages were overcome with the discovery of a fast computational algorithm for evaluating the MVSE. Musicus [6] discovered that the MVSE denominator could be evaluated efficiently by exploiting the structure of the inverse Toeplitz

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autocorrelation matrix. The Toeplitz inverse can be formulated in terms of triangular Toeplitz matrix products with matrix elements composed of autoregressive (AR) parameters [7]. However, in practice, the autocorrelation matrix is unknown and only data samples are available, which normally requires that the MVSE be evaluated by first estimating the autocorrelation sequence (ACS) from the data samples. We consider here alternative formulations of the MVSE in terms of least-squares minimization of the linear prediction (LP) variance, as discussed in Sections 2.3 and 2.4. The denominators of these LS-based MVSEs in the covariance and modified covariance LP cases (refer to Eqs. (11) and (14), respectively) are functions of *near-to-Toeplitz* matrix inverses. The inverses of these *near-to-Toeplitz* matrices can also be formulated in terms of triangular Toeplitz matrix products, with matrix elements composed of the LP algorithm parameters rather than the AR parameters. The remainder of this section will introduce the derivation of the two LS-based MVSEs in the covariance and modified covariance LP cases. Sections 4 and 5 will provide the fast computational solutions for the LS-based MVSE that exploit the inverse matrices of the *near-to-Toeplitz* LS-based data product matrices. The fast algorithm of the covariance LS-MVSE has been reported by the authors of this paper [8].

We consider two types of LP approaches to generate the LP parameters: (1) the covariance LP approach utilizes separate forward and backward LP estimation and (2) the modified covariance LP approach employs a combined forward and backward LP estimation. This leads to two LS-based MVSE expressions that involve inverses of *near-to-Toeplitz* products of data matrices. In this paper, we show that the *near-to-Toeplitz* matrix inverses can be formulated in terms of triangular Toeplitz matrix products composed of LP parameters. Once the LP parameters are estimated, these can then be applied to the *near-to-Toeplitz* matrix inverse expressions that lead to fast computational algorithms for evaluating the covariance LS-based MVSE and the modified covariance LS-based MVSE spectral density expressions. The two new fast MVSE algorithms have spectral detail performances that exceed the traditional autocorrelation-based (ACS-based) MVSE, yet are just as computationally efficient as the ACS-based MVSE. The computational complexities of the two fast solutions of the LS-based MVSE are proportional to p^2 where p is the estimator order in this paper, with memory storage requirements proportional to p, versus a p^3 computational efficiency and p^2 storage requirement if solved by using a direct inversion approach. In addition, the two fast algorithms generate all intermediate order LP parameters from order 0 to order p, which means all MVSE spectral plots from order 0 to order p can be evaluated without recomputing the LP parameters of all orders.

This paper is organized as follows. Section 2 provides the basis for the ACS-based MVSE and the two LS-based MVSEs. This section will also illustrate that the new LS-based MVSE in either the covariance case or the modified covariance case has spectral resolution performance advantages when compared with the classical periodogram, the MVSE that uses either the AR Yule–Walker algorithm or the MVSE that uses the lattice-based Burg algorithm (Burg-lattice) to estimate the AR parameters. Section 2 will also illustrate that (1) the ACS-based MVSE depends on the inverse of the autocorrelation matrix with Toeplitz structure, (2) the LS-based MVSE in the covariance case depends on the inverse of a *near-to-Toeplitz* product matrix formed from data matrices, and (3) the LS-based MVSE in the modified covariance case also depends on the inverse of an expanded *near-to-Toeplitz* product matrix formed from data matrices. In Section 3, we will summarize the Musicus fast algorithm for the ACS-based MVSE, which exploits the inverse of the Toeplitz autocorrelation matrix in terms of products of triangular Toeplitz matrices and then applies a FFT to evaluate the denominator of the ACS-based MVSE. In Sections 4 and 5, we will present the derivations of the new fast algorithms based on exploiting their special inverse structures for the *near-to-Toeplitz* product matrix formed from data matrices. Finally, Section 6 will summarize the contributions described in this paper.

2. Autocorrelation-based MVSE and least-squares-data-based MVSE

2.1. Enhanced performance of least-squares-data-based MVSE over ACS-based MVSE

The performances of the two new fast algorithms for the LS-based MVSE in the covariance case and in the modified covariance case developed in this paper are illustrated in Fig. 1. In this experiment, a test case is generated as a combination of both narrow-band and wide-band signals. A 64-complex-point simulated Doppler radar data set [9] is used to test the resolution capability of the two new fast solutions for the LS-based MVSE. The Doppler frequency F_d is proportional to (Vf_c/c) [10], where V is the radial velocity of an object moving toward/away from a radar, f_c is the carrier frequency (assumed here as 10 GHz), and c is the light speed (3×10^8 m/s). The true spectrum of the test data, calculated by analytical means, is illustrated in Fig. 1(a). The frequency axis is expressed as a fraction of the sampling frequency ($f_s = 2500$ Hz in this simulation). The five stems at $-0.3, -0.1, 0.2, 0.21$ and 0.4 in the fraction of sampling frequency axis simulate five aircraft flying at different radial velocities toward or away from the radar. The two closest stems with highest power at 0.2 and 0.21 fraction of sampling frequency are used to test the resolution capability of an estimator. The short stem at 0.4 fraction of sampling frequency with lower power is used to test if an estimator can pick out a weak signal component among strong noise content. The colored noise process is created by passing white noise through a filter (with a truncated cosine shaped frequency response) to simulate typical low frequency clutter effects due to wind (10–40 m/h) [11], cloud motions (0–100 m/h) [12], flying birds (0–100 m/h) [11] and moving traffic on a road (0–70 m/h) [13].

The estimator spectral responses produced from the 64-complex-samples are illustrated in Fig. 1 for order $p = 12$. In Fig. 1(b), the Nuttall window has been used to