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A fast algorithm for the simulation of arterial pulse waves

Tao Du [a], Dan Hu [a,*], David Cai [a,b,c]

[a] Department of Mathematics, Institute of Natural Sciences, and MOE-LSC, Shanghai Jiao Tong University, Shanghai, China
[b] Courant Institute of Mathematical Sciences and Center for Neural Science, New York University, New York, USA
[c] NYUAD Institute, New York University Abu Dhabi, PO Box 129188, Abu Dhabi, United Arab Emirates

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ABSTRACT

One-dimensional models have been widely used in studies of the propagation of blood pulse waves in large arterial trees. Under a periodic driving of the heartbeat, traditional numerical methods, such as the Lax–Wendroff method, are employed to obtain asymptotic periodic solutions at large times. However, these methods are severely constrained by the CFL condition due to large pulse wave speed. In this work, we develop a new numerical algorithm to overcome this constraint. First, we reformulate the model system of pulse wave propagation using a set of Riemann variables and derive a new form of boundary conditions at the inlet, the outlets, and the bifurcation points of the arterial tree. The new form of the boundary conditions enables us to design a convergent iterative method to enforce the boundary conditions. Then, after exchanging the spatial and temporal coordinates of the model system, we apply the Lax–Wendroff method in the exchanged coordinate system, which turns the large pulse wave speed from a liability to a benefit, to solve the wave equation in each artery of the model arterial system. Our numerical studies show that our new algorithm is stable and can perform ~ 15 times faster than the traditional implementation of the Lax–Wendroff method under the requirement that the relative numerical error of blood pressure be smaller than one percent, which is much smaller than the modeling error.

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1. Introduction

In Traditional Chinese medicine, the temporal profile of blood pressure is believed to be a useful indicator of an individual's physiological condition [1,2]. Strong correlations have been observed between the blood pulse profiles and diseases, such as hypertension [3–5], arteriosclerosis [6,7], and diabetes mellitus [8]. Certain blood pulse profiles in these diseases are believed to be resulted from abnormality in geometric structures and elastic properties of the arterial system [9–13]. In computational modeling, one-dimensional models have been shown to be able to capture the propagation of blood pulse waves and the profile of the blood pressure in large arterial trees [14–28]. The computational modeling has provided an important approach to the study of relations between abnormal blood pressure profiles and diseases [22,23,26,29].

In many applications, a large amount of simulations of the one-dimensional model may be necessary. For example, different diseases can lead to different types and varying degrees of alterations in the geometric structures and elastic properties of the arterial tree. To study the relation between these changes and the resulting blood pulse profiles [22,23,26,29], there is a need to use a large number of numerical simulations to explore all possible dynamics of the arterial tree

* Corresponding author.
E-mail addresses: duty1998@sjtu.edu.cn (T. Du), hudan80@sjtu.edu.cn (D. Hu), cai@cims.nyu.edu (D. Cai).

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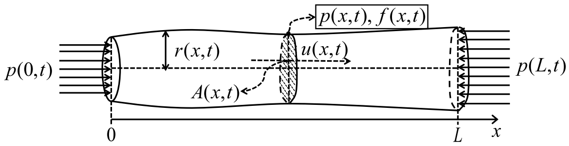

Fig. 1. An axisymmetric vessel with radius $r(x,t)$ and fixed length L. The direction of the average blood velocity $u(x,t)$ is along the axis of the vessel. The viscous effect $f(x,t)$ arises from the shear stress on the vessel wall.

brought about by those changes. In general, it is difficult to obtain all detailed information of the physical properties of the arterial tree as well as the inflow information at the heart. To quantify this type of uncertainty of dynamical models and the accuracy of computed physiological properties in virtual populations, it is also necessary to perform a great number of simulations with a broad range of parameters [21,30,31]. Furthermore, following the idea of the Traditional Chinese medicine, the inverse problem of gauging the blood pulse profiles to apprehend clinical changes associated with properties of the arterial system is of particular interest. To tackle this problem successfully, it would also entail a large number of simulations of the forward problem. Clearly, a fast algorithm for solving the one-dimensional model would greatly facilitate these studies.

Traditional numerical methods, such as the Lax–Wendroff method [32], are widely used in solving the one-dimensional model [15,29,33–37]. However, due to the large speed of blood pulse waves (*e.g.*, $\sim 1 \times 10^3$ cm/s) and short lengths of vessels (*e.g.*, the length of hepatic artery is ~ 1 cm, see Table 1) [5,38], the time step required by the CFL condition [39] for the Lax–Wendroff method is usually very small (*e.g.*, $\sim 10^{-4}$ s). The requirement of small time step by the CFL condition also presents a strong constraint in the implementation of other methods for solving the one-dimensional model, including the MacCormack scheme [28,40], the upwind scheme [41], the finite element method [24,25], the Galerkin method [20–22,26], and the domain-decomposition method [27]. Furthermore, because the heartbeat is approximately periodic, one usually seeks periodic solutions of the one-dimensional model for blood pulse waves. This often requires one to evolve the model for sufficiently long time (~ 10 periods) to obtain an asymptotic periodic solution. As demanded by the small time step and long simulation time, the simulation can become rather expensive.

In this work, we develop a fast algorithm for the simulation of the one-dimensional model of blood pulse waves. First, we reformulate the model in terms of a set of Riemann variables. Then we exchange the temporal and spatial coordinates and apply the Lax–Wendroff method in the exchanged coordinate system to solve the wave equations in each vessel. Finally, in order to enforce the boundary conditions at the inlet, the outlets, and the bifurcation points of the arterial tree, we derive a new form of the boundary conditions, which allows us to design a stable iterative method. The new algorithm can overcome the constraint of the CFL condition in the case of large wave speed. In fact, in our algorithm, the large wave speed is no longer a liability — instead, it has become an advantage. As a result, the computational cost of our algorithm reduces significantly. Under the requirement that the numerical error of blood pressure be smaller than one percent, which is already much smaller than the error of the model, our algorithm yields ~ 15 times of speedup compared to the traditional scheme [15,33].

2. One-dimensional model for arterial pulse waves

The propagation of blood pulse waves in an arterial tree includes two processes — the interaction between the blood flow and the elastic vessel walls in single vessels and the wave transmission and reflection at the vessel bifurcation points (or junctions). The first one is described by a system of partial differential equations and the second one is described by the boundary conditions at the bifurcations of the arterial tree. The one-dimensional model for blood pulse waves has been studied in the previous works [4,5,15,16,19,42]. In this section, we briefly review the model and reformulate the system using two sets of Riemann variables, which are used to facilitate the design of our algorithm.

2.1. Blood pulse wave in a single vessel

As is in the previous works [4,5,15,16,19,42], a blood vessel is regarded as an axisymmetric compliant tube with fixed length L and variable radius $r(x,t)$, where t is the temporal coordinate, x is the spatial coordinate along the central axis of the vessel, as shown in Fig. 1. The blood is assumed to be an incompressible Newtonian fluid. Under long wavelength approximation, the conservation of momentum and mass of the blood flow in the tube yields

$$\begin{aligned} u_t + u u_x + \frac{p_x}{\rho} &= \frac{f}{\rho}, \\ A_t + A u_x + u A_x &= 0, \end{aligned} \tag{1}$$

where $u(x,t)$ is the average blood velocity over the cross section at x, $p(x,t)$ is the average pressure on the cross section, ρ is the constant density of the blood, $A(x,t) = \pi r(x,t)^2$ is the cross-sectional area, and $f(x,t)$ represents the viscous effect on the vessel wall

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