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Reduction of the total execution time to achieve the optimal *k*-node reliability of distributed computing systems using a novel heuristic algorithm

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Abstract

A distributed computing system is a collection of processor–memory pairs connected by communication links. A *k*-node set is a subset of total nodes in a distributed computing system. A *k*-node set with capacity constraint is a *k*-node set that possesses sufficient node capacity. Because computing the reliability of a distributed computing system is generally an NP-hard problem, an adequate *k*-node set with a given capacity constraint must be determined by an effective algorithm with an approximate reliability. Relatively few investigations, namely an exact method and a *k*-tree reduction method, have examined *k*-node reliability optimization with capacity constraint. Such investigations either spent an exponential time or rarely obtained an optimal solution. Therefore, in this work, we present a novel heuristic algorithm to reduce the computational time and deviation from an exact solution. The proposed algorithm has simple independent steps, including selection of *k*-node sets according to a node's weight or a link's weight. The number of selected *k*-node sets is either one or two, thereby spending less time to compute the reliability of *k*-node sets. Computational results demonstrate that the proposed algorithm is more effective and provides a better solution for a large distributed computing system than those in previous investigations.

Keywords: Distributed computing systems (DCS); Reliability optimization; *k*-Node reliability (KNR)

1. Introduction

Recent developments in computer networking and low cost computational elements have led to increasing interest in distributed computing systems (DCS). DCS, a collection of processor–memory pairs connected by communication links, is logically integrated by a distributed communication network. The communication subnet may be a geographically dispersed collection of communication processors or a local area network [1–3]. The numerous merits of using DCS include improved resource sharing, enhanced fault tolerance and high reliability.

The economic benefits of resource sharing largely account for the importance of DCS. A DCS focuses on providing efficient communication among various nodes, thereby increasing their reliability and making their service available to more users [4]. Designing such systems must consider system reliability which heavily relies on the topological layout of communication links [5–7].

The topology of a network can be characterized by a linear graph. These network topologies can be characterized by their network reliability, message-delay, or network capacity. The performance characteristics depend on many properties of the network topology [8–13]: the number of ports at each node (degree of a node) and the number of links. The number of links directly impacts the system reliability.

Previous literature provides reliability optimization models of DCS that optimize source to destination reliability, *k*-out-of-*n* systems reliability and overall system reliability [7,14,15]. Two reliability optimization models have been presented in [16].

As defined, *k*-node reliability is the probability that nodes in the *k*-node set (subset of the set of processing elements) in the DCS are connected. The exact method (EM) [17] and the *k*-tree reduction method [18] have examined *k*-node reliability optimization with capacity constraint. These either spent an exponential time or barely obtained an optimal solution.

This work focuses mainly on how to compute nearly maximum system reliability objectives with capacity

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constraint. The fact that computing the reliability of DCS is a NP-hard problem explains why an adequate solution must be derived in a very short time. More specifically, deriving a solution with an exact solution as possible is of primary concern. Therefore, this work computes the reliability of a subset of the set of processing elements such that the reliability is maximized and the specified capacity constraint is satisfied.

2. Problem description

In this section, we described the problem addressed herein to clarify our research objectives.

2.1. Notations and definitions

The following notations and definitions will be used.

$c(G_k)$, $c(G'_k)$ the sum of capacity of the k-node set G_k and G'_K, respectively
$c(v_i)$ the capacity of the i^{th} node.
$d(v_i)$ the number of links connected to the node v_i
e the number of links in G, $e = |E|$
$e_{i,j}$ an edge represents a communication link between v_i and v_j, $e_{i,j} \in E$
n the number of nodes in G, $n = |V|$.
$p_{i,j}$ the probability of success of link $e_{i,j.}$
$q_{i,j}$ the probability of failure of link $e_{i,j.}$
v_i the i^{th} processing element or the i^{th} node, $v_i \in V$
$w(v_i)$ the weight of the i^{th} node
$w(e_{i,j})$ the weight of link $e_{i,j}$
$y_{i,j}$ the number of path which length is 2 between v_i and v_j
$C_{\text{constraint}}$ total capacity limit in a DCS.
$G = (V,E)$ an undirected DCS graph where V denotes a set of processing elements, and E represents a set of communication links
G_k, G'_K the graph G with the set K of nodes specified, and $|K| \geq 2$, G_k, $G'_K \subset G$
P the link reliability matrix where $P(i,j) = P(j,i) = p_{i,j}$ if $e_{i,j}$ exists in G, $p(i,j) = p(j,i) = 0$, otherwise for $i,j = 1,2,\ldots,n$
$R(G_k)$, the reliability of the k-node set
$R(G'_K)$ solution of a DCS graph G
$V_{\text{adj}(Gk)}$, $V_{\text{adj}(G'_k)}$ a set of nodes which are adjacent to any node of G_k and G'_K, respectively

Definition 1. A k-node reliability(KNR) is defined as the probability that a specified set K of nodes is connected (where K denotes a subset of the set of processing elements).

Definition 2. A node v_i is directly connected to a set G_k of nodes if and only if there is a link between v_i and a node in G_k.

Definition 3. The number of reliability computation (NRC) is the number of k-node sets whose reliability should be computed.

2.2. Problem statements

Bi-directional communication channels operate between processing elements. A distributed computing system can be modeled by a simple undirected graph. For a topology of the DCS with four nodes, say $V = \{v_1, v_2, v_3, v_4\}$, and five links, say $E = \{e_{1,2}, e_{1,3}, e_{2,3}, e_{2,4}, e_{3,4}\}$, there are many sunsets of nodes. A set, K, is a subset of the DCS which includes some nodes of the given node set V. The KNR is the probability that a specified set K of nodes is connected, where K denotes a subset of set of processing elements. For example, $K = \{v_1, v_2, v_3\}$ is selected in the DCS with bridge topology. The reliability of the set, K, can be computed by means of a sum of mutually disjoint terms [19].

$$R(G_k) = p_{1,2}p_{1,3}q_{2,3}q_{2,4} + p_{1,2}p_{1,3}q_{2,3}p_{2,4}q_{3,4} + p_{1,2}q_{1,3}p_{2,3}q_{2,4}$$
$$+ p_{1,2}q_{1,3}p_{2,3}p_{2,4}q_{3,4} + p_{1,3}p_{2,3}q_{2,4} + p_{1,3}p_{2,3}p_{2,4}q_{3,4}$$
$$+ p_{1,2}q_{1,3}p_{2,4}p_{3,4} + p_{1,3}p_{2,4}p_{3,4}$$

Assume that probability of $p_{1,2}, p_{1,3}, p_{2,3}, p_{3,4}$ is 0.95, 0.94, 0.93, 0.92, and 0.91, respectively. Then $R(G_k) = 0.9958148$.

A k-node reliability problem can be characterized as follows:
Given
 Topology of a DCS.
 The reliability of each communication link.
 The capacity of each node.
 A set of data files.
Assumption
 Each node is perfectly reliable.
 Each link is either in the working (ON) state or failed (OFF) state.
Constraint
 The total capacity of data files to be allocated.
Goal
 To select a specified set K of nodes in a DCS to which to allocate data files, by doing so, k-node reliability is adequate under capacity constraint.

Reliability optimization can be defined in the maximum reliability for computing a given task under some constraints. For a given task, its reliability can be computed as $R_1, R_2, \ldots, R_x$ for x conditions. By doing so, the reliability optimization for the task is the maximal reliability in $R_1, R_2, \ldots, R_x$. The heuristic algorithm involves obtaining