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Long-term Nash equilibria in electricity markets

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ABSTRACT

In competitive electricity markets, companies simultaneously offer their productions to obtain the maximum profits on a daily basis. In the long run, the strategies utilized by the electric companies lead to various long-term equilibria that can be analyzed with the appropriate tools. We present a methodology to find plausible long-term Nash equilibria in pool-based electricity markets. The methodology is based on an iterative market Nash equilibrium model in which the companies can decide upon their offer strategies. An exponential smoothing of the bids submitted by the companies is applied to facilitate the convergence of the iterative procedure. In each iteration of the model the companies face residual demand curves that are accurately modeled by Hermite interpolating polynomials. We introduce the concept of meta-game equilibrium strategies to allow companies to have a range of offer strategies where several pure and mixed meta-game Nash equilibria are possible. With our model it is also possible to model uncertainty or to generate price scenarios for financial models that assess the value of a generating unit by real options analysis. The application of the proposed methodology is illustrated with several realistic case studies.

1. Introduction

In a restructured electric environment, electricity markets represent an effective system for the purchase and sale of electricity. Many electricity markets in the world have been established applying a pool-based auction paradigm. In an auction-based day-ahead market [1–3], the market operator processes the bid information provided by the producers and consumers and aggregates this information creating hourly offer and demand curves, respectively. Both producers and consumers bid with the target of maximizing their profits. Once the bids are submitted, a market-clearing algorithm matches the production and demand curves producing a series of hourly prices and accepted quantities.

Searching for possible market equilibria is a desirable objective both for market participants and regulators. For participants, because an equilibrium shows long-term bidding strategies of their rivals; for regulators, because market power monitoring and corrective measures are possible. The knowledge of long-term equilibria represents a valuable tool for electric companies to implement their bidding strategies. In addition, electric companies need to know what their offer strategy should be against every possible offer strategy of their competitors for long time periods. To find market equilibria, it is necessary: (i) to simulate how participants generate their offers and bids, (ii) to establish a market-clearing procedure, and (iii) to identify plausible equilibria. Steps (i) and (ii) are possible by means of optimization techniques [4,5] and step (iii) relies on the concept of Nash equilibrium [6,7].

Existing literature related to equilibrium models in electricity markets mainly focuses either on Nash–Cournot models or on supply function equilibria (SFE) models. The Nash–Cournot equilibrium concept has been applied to calculate equilibria in multi-period settings either by iterative simulation, as in [8,9], or by mathematical optimization methods, as in [10,11]. SFE models have been also applied since its introduction by the seminal paper from Klemperer and Meyer [12]. One of its first applications was in the British spot market by Green and Newbery [13] and subsequent studies by Baldick [14], among others [15], where uncertainty is considered in their approach [16]. Finding Nash equilibria by simulation is also possible combining mathematical optimization and game theory. Game theory simulators are closely connected with market equilibrium models, several works have tackled the use of game theory models and/or agent-based models within electricity markets' simulators [17,18]. In addition, other types of equilibria, such as Forchheimer or Bertrand are studied in an electricity market simulator framework [19].

One of the main problems of iterative market simulation models is the lack of convergence [8]. This issue can be interpreted considering that lower prices in an iteration result in smaller offered quantities in the next iteration and vice versa and no stable solution can be found. We have solved this problem using an exponential

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smoothing scheme of the energies bid by the companies weighting past energy bids with exponentially decreasing weights.

Additionally, the calculation of the residual demand curves as a result of the subtraction of the companies' bids from the total demand is accurately represented by means of a Hermite polynomial approximation.

Therefore, to have a realistic equilibrium model of a pool-based electricity market we have developed in this paper: (i) an iterative electricity market bidding model and a market-clearing model that finds Nash equilibria strategy following a SFE approach when each of the companies selects a particular offer, (ii) the concept of *offer parameter* to bid in the market, (iii) a *meta-game* equilibrium model that finds all the equilibria for a range of strategies of the companies, (iv) a Hermite polynomial approximation of the residual demand curves faced by the companies, and (v) an exponential smoothing procedure to achieve convergence in the iterative bidding.

The paper is organized as follows. Section 2 presents the iterative market equilibrium model including the notion of offer parameter. Section 3 introduces the concept of meta-game equilibrium. Section 4 presents several case studies. In Section 5, extensions to the model are described, and conclusions are outlined in Section 6.

2. Market equilibrium iterative model

To model a general pool-based electricity market we consider that the agents offer and/or bid using discrete blocks. Although the demand is generally assumed inelastic in actual markets, we have modeled it using a step-wise function. Both uncertainty and network modeling are disregarded since we aim to find realistic long-term equilibria in electricity markets without serious congestion problems. Uncertainty could be included in the model for demands, prices and bids but it will be part of future research. For a more detailed economic dispatch model under uncertainty and step-wise offers see [20], or, with a generating company focus, see [21].

It is assumed that a unique price exists resulting from the matching of generation and demand exists for the entire market. As regards to the market-clearing algorithm, we only consider the upper and lower bounds of the produced power, disregarding other constraints such as ramps, start-up and shut-down times. The production costs of the generating units are modeled using linear functions. Note that the aforementioned constraints and the ones derived from the inclusion of the network produce highly complex models that can lead to exceedingly long running times for long-term simulations.

Our focus is to produce an equilibrium based upon an iterative method, in which the agents are considered as rational and compete to obtain the maximum possible profits. In the proposed model each company optimizes the power to supply the day-ahead market using different offer strategies.

2.1. Demand model

We assume that the demand can be elastic by using discrete demand bidding blocks.

Since in long-term simulations (1 year or more), market bidding and subsequent equilibria involve 24 × 365 = 8760 periods that would mean that the level of detail may be cumbersome to deal with. A more logical approach would be to split the days into several hourly periods. For example, for 1 day, hours 1–8 may correspond to the first period, and hours 9–24 to the second. In Fig. 1 we observe that for a week with 168 h, the number of periods is 6, reducing the problem by 28. We have used the following periods in the model:

- Period 1: Monday, hours 1–8.
- Period 2: Monday, hours 9–24.
- Period 3: Tuesday to Friday, hours 1–8.
- Period 4: Tuesday to Friday, hours 9–24.
- Period 5: Saturday, hours 1–24.
- Period 6: Sunday, hours 1–24.

2.2. Market-clearing algorithm

The market-clearing algorithm used determines the power produced by each generating unit so that the social welfare is maximized and the demand is met during all the periods.

Social welfare objective function:

The objective function to maximize is the social welfare for the entire time horizon. It is defined as the area between the aggregate bid curve and the aggregate offer curve, both ordered by decreasing and increasing prices, respectively. Maximizing this function is equivalent to minimizing the production cost if the demand is inelastic. The mathematical expression of the social welfare is:

$$\max_{p^G_{jtb}, p^D_{dth}} \sum_{t=1}^{N_T} \left[\sum_{d=1}^{N_D} \sum_{h=1}^{N_H} p^D_{dth} \lambda^{D\text{-}bid}_{dth} - \sum_{j=1}^{N_J} \sum_{b=1}^{N_B} p^G_{jtb} \lambda^{G\text{-}offer}_{jtb} \right] \quad (1)$$

Constraints:

The constraints that have to be met are described as follows:

- The first block offered by each generating unit must be equal to its lower production bound:

$$p^G_{jt1} = \underline{P}_j \quad \forall t \in \Omega_T, \forall j \in \Omega_J \quad (2)$$

- The power generated by any generating unit in any given period is the summation of its corresponding production blocks. This power must be lower than or equal to the upper production bound of the unit:

$$\sum_{b=1}^{N_B} p^G_{jtb} = p^G_{jt} \le \overline{P}_j \quad \forall t \in \Omega_T, \forall j \in \Omega_J \quad (3)$$

- The power produced by each block of a generating unit is limited by the size of the block:

$$0 \le p^G_{jtb} \le \overline{p}^G_{jtb} \quad \forall t \in \Omega_T, \forall j \in \Omega_J, b = 2, \ldots, N_B \quad (4)$$

- The power consumed by any demand in any given period is the summation of its corresponding consumption blocks:

$$\sum_{h=1}^{N_H} p^D_{dth} = p^D_{dt} \quad \forall t \in \Omega_T, \forall d \in \Omega_D \quad (5)$$

- The power consumed by each block of a demand is limited by the size of the block:

$$0 \le p^D_{dth} \le \overline{p}^D_{dth} \quad \forall t \in \Omega_T, \forall d \in \Omega_D, h = 1, \ldots, N_H \quad (6)$$

- The production must match the demand in every period:

$$\sum_{d=1}^{N_D} \sum_{h=1}^{N_H} p^D_{dth} = \sum_{j=1}^{N_J} \sum_{b=1}^{N_B} p^G_{jtb} \quad \forall t \in \Omega_T. \quad (7)$$

2.3. Generating companies quantity optimization model

To model a pool-type day-ahead market we consider thermal units, where all of them maximize their profits. Thermal units encompass both fossil-fueled and nuclear units. The latter ones can be seen as thermal units with very low production costs. The objective function of a generating company is obtained subtracting the