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Existence of Nash equilibrium in games with a measure space of players and discontinuous payoff functions ☆

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Abstract

Balder's [6] model of games with a measure space of players is integrated with the line of research on finite-player games with discontinuous payoff functions which follows Reny [47]. Specifically, we extend the notion of continuous security, introduced by McLennan, Monteiro & Tourky [38] and Barelli & Meneghel [9] for finite-players games, to games with a measure space of players and establish the existence of pure strategy Nash equilibrium for such games. A specification of our main existence result is provided which is ready to fit the needs of applications. As an illustration, we consider several optimal income tax problems in the spirit of Mirrlees [40] and use our game-theoretic result to show the existence of an optimal income tax in each of these problems.

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1. Introduction

The line of research initiated by Dasgupta & Maskin [20] and continued, amongst others, by Reny [47] has been successful in obtaining equilibrium existence results for finite-player games with discontinuous payoff functions. In this paper we extend this approach to the context of generalized games with a measure space of players, a class of games first considered by Schmeidler [53], the state of the art now set by Balder [6]. In particular, concerning existence of Nash equilibrium, we bring the branch of game theory dealing with games with a measure space of players on par with that dealing in a systematic way with games with discontinuous payoff functions.[1,2]

Apart from obtaining a unification of important recent game-theoretic results on a general level, a motivation for our analysis is that several economic problems which are addressed in the literature can be modeled as games with a continuum of players, but where payoff functions need not be continuous, and need not even satisfy the assumptions in [6]. As an example, we will consider a version of Mirrlees's [40] model of optimal taxation (see Section 9).

Our approach to deal with discontinuous payoff functions in the setting of games with a measure space of players is based on the notion of multiply security, which was developed in the context of finite-player games by McLennan, Monteiro & Tourky [38]. More precisely, we take a version of this notion, called continuous security, which was introduced by Barelli & Meneghel [9], and adapt it to the particular measurability needs arising when there may be a continuum of players. We remark that the notion of multiply security generalizes that of better-reply security, which was introduced in the pioneering paper of Reny [47].

Based on the notion of continuous security, our result covers, in particular, games where, as in [6], payoff functions are assumed to be upper semi-continuous and the value functions of the players are assumed to be lower semi-continuous.[3] In fact, when value functions are assumed to be lower semi-continuous, it covers games with payoff functions that are merely weakly upper semi-continuous (as defined in [14]).

In addition to the pure strategy existence result of Balder [6], our result extends that of Khan & Sun [31]. The approaches of Balder [6] and Khan & Sun [31] differ in the way how a convexifying effect of aggregation is ensured to deal with a continuum of negligible players with non-convex action sets. In [6] it is assumed in this respect that the payoff of each player may depend on the actions of the other players only through a finite-dimensional vector of summary statistics. In [31], on the other hand, the entire distribution of the actions of the players may matter for payoffs, but the non-atomicity hypothesis on the measure on the space of players is strengthened. Our result will show that the same strengthening of non-atomicity still allows to obtain pure-strategy Nash equilibria, with non-convex action sets and payoff dependence modeled as in [31], when, differently to Khan & Sun [31], payoff functions need not be continuous.

The paper is organized as follows. We present a motivating example in Section 2. Some notation and terminology is introduced in Section 3. In Section 4 we present the general model and our notion of continuous security. Section 5 contains the statements of our main existence

[1] Further papers on existence of equilibrium with a measure space of players include Barelli & Duggan [8], Carmona & Podczeck [17], Khan [29], Khan, Rath & Sun [30], Khan & Sun [31], Martins da Rocha & Topuzu [36], Mas-Colell [37], Páscoa [42], and Rath [46].

[2] Recent papers on finite-player games with discontinuous payoff functions include Bagh & Jofre [2], Balder [7], Bich [11], Bich & Laraki [12], Carmona [15,16], de Castro [21], Nessah [41], Prokopovych [44,45], Reny [48–50].

[3] See Section 4.1 for the formal definition of the value function of a player.