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An undominated Nash equilibrium for voting by committees with exit

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Abstract

We consider the problem of a society whose members choose, with a voting by committees, a subset of new members from a given set of candidates. After knowing the elected candidates, former members may decide to either stay or exit the society. We analyze the voting behavior of members who take into account the effect of their votes not only on the elected candidates, but also on the final composition of the society. For additive and monotonic preferences with dichotomous bads we construct a strategy profile that is an undominated pure strategy Nash equilibrium of the induced voting game.
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1. Introduction

Societies use voting rules to make decisions. The elections of representatives in democratic societies, the public positions taken up by political parties on different issues, or the admission of new members in a society are some examples of this. For this last example, Barberà et al. (1991)

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consider the problem where a finite set of members who originally make up a society has to decide which candidates, to be chosen from a given set, will be elected to become new members of the society. They assume that former members of the society cannot leave it as a result of its change in composition. But often, the entrance of new members triggers the exit of former ones. For the static setting where members *cannot* leave the society, they characterize *voting by committees* as the class of strategy-proof and onto social choice functions whenever members' preferences over subsets of candidates are either separable or additively representable. However, strategy-proofness becomes too strong whenever the mentioned evolution of the society is explicitly considered. The aim of this paper is to study the strategic behavior of members by means of the analysis of undominated pure strategy Nash equilibria in a complete information setting where all members' preferences are common knowledge.[1]

Three lines of research have already focused on the analysis of strategic equilibria under complete information. A first one considers a society that, during a fixed and commonly known number of periods, may admit in each period a subset of new members. Within this dynamic setup, an interesting issue arises: voters, at earlier stages, vote not only according to whether or not they like a candidate but also according to their tastes concerning future candidates. Barberà et al. (2001) study the particular case where members have dichotomous preferences (candidates are either friends or enemies) and the voting rule used by the society is quota one (it is sufficient to receive one vote to be elected). They identify and study (subgame perfect and trembling-hand perfect) *equilibria* where members exhibit, due to the dynamics of the game, complex strategic voting behavior. Granot et al. (2002) study a similar model with expulsion; current members of the society have to decide each period whether to admit by unanimity new members into the society *and* whether to expel current members by others' unanimity. They study equilibria for different protocols which depend on whether the expulsion decision has to be taken each period either simultaneously with, before, or after the admission decision.

In a second line of research, a set of voters and a set of candidates (which may overlap) must select a representative candidate (or a subset of them). The key issue this literature addresses is the incentives of candidates, given a particular voting rule (how voters choose a candidate or a subset of candidates), to enter or exit the election in order to strategically affect the outcome of the rule. By imposing some independence conditions and an internal "stability condition" (the losing candidates must not have an incentive to drop out of the election) they prove that the class of voting rules immune to this strategic manipulation is only composed of dictatorial rules.[2]

In this paper we contribute on a third line of research by considering explicitly the possibility that, in the Barberà et al. (1991) setup members who originally conform a society have the option to leave it voluntarily. In Berga et al. (2004) we showed that the unique social choice function that is still strategy-proof, stable, and satisfies founders' sovereignty on the set of candidates is the voting by committees that requires unanimity for the entrance of each candidate.[3] The dynamic aspect of this decision is hidden in the general formulation of the

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[1] A more difficult line of research would consist of considering incomplete information and concentrating on the analysis of Bayesian equilibria, for example. But this is outside the scope of this paper.

[2] See Dutta, Jackson, and Le Breton (2001) for single-valued voting rules, and Ehlers and Weymark (2003), Eraslan and McLennan (2004), and Rodrıguez-Álvarez (2006) for multi-valued voting rules.

[3] Stability requires the exit to be voluntary; that is, for any preference profile the social choice function has the property that all members belonging to the final society want to stay (internal stability) and all members who do not belong to the final society do not want to belong (external stability). Founders' sovereignty on the set of candidates requires that candidates that are good for all members have to be admitted to the society and candidates that are bad for all members cannot be admitted.