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Feedback Nash equilibria for non-linear differential games in pollution control

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Abstract

Dynamic problems of pollution and resource management with stock externalities often require a differential games framework of analysis. In addition they are represented realistically by non-linear transition equations. However, feedback Nash equilibrium (FBNE) solutions, which are the desired ones in this case, are difficult to obtain in problems with non-linear-quadratic structure. We develop a method to obtain numerically non-linear FBNE for a class of such problems, with a specific example for shallow lake pollution control. We compare FBNE solutions, by considering the entire equilibrium trajectories, with optimal management

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and open-loop solutions, and we show that the value of the best FBNE is in general worse than the open-loop and optimal management solutions.

JEL classification: Q25; C73; C61

Keywords: Differential games; Pollution control; Non-linear feedback Nash equilibrium solution; Abel equation; Open-loop; Optimal management

1. Introduction

Many pollution control models have a similar format. Emissions as a by-product of production or consumption accumulate into a stock of pollutants, which is damaging in some way. Production of energy with fossil fuels, for example, releases CO_2 and this contributes to the stock of greenhouse gases, which may cause damage through climate change. To take another example, agricultural activities release phosphorus into lakes, and the resulting stock of phosphorus causes a loss of ecological services, provided by these lakes. Resource extraction problems also have a similar format, if the stock of the resource has some direct utility besides the option to extract in the future. For example, a forest may have an amenity or a biodiversity value besides the option to extract wood. Another important characteristic of these problems is that the damage usually is a public bad. This implies that the optimal control models, which can be set-up to handle the trade-offs above, have stock externalities, which turn the framework of analysis into a differential game. The techniques developed in this paper apply to this general class of problems but in order to keep the presentation transparent, the paper focuses on pollution control problems and, more specifically, on the eutrophication of lakes.

Differential games have been extensively studied during the recent decades to analyze economic problems in areas such as industrial organization, resource and environmental economics or macroeconomic policy. The solution concept that is most often used is the open-loop Nash equilibrium (OLNE), where controls only depend on time (and the initial state of the system). As it is well known, the OLNE is weakly time-consistent but not strongly time-consistent (Başar, 1989): it does not possess the Markov perfect property and is not robust against unexpected changes in the state of the system. Therefore, the feedback Nash equilibrium (FBNE) is a more satisfactory solution concept. It is derived in a dynamic programming framework, so that controls depend on time and state, and the solution is Markov perfect by construction. However, solutions are usually very difficult to derive. It is straightforward to find linear feedback equilibria for problems with linear dynamic systems and quadratic objectives, but two types of possible non-linearities complicate matters considerably. The first one is the possibility of *strategic* non-linearities. Tsutsui and Mino (1990) have shown that non-linear equilibria may exist for a linear-quadratic formulation of the dynamic duopoly with sticky prices and their result has later been applied to other problems, such as the problem of