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Games and Economic Behavior 63 (2008) 328–340
GAMES and Economic Behavior
www.elsevier.com/locate/geb

The number of pure Nash equilibria in a random game with nondecreasing best responses

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Received 17 August 2004
Available online 4 November 2007

Abstract

We randomly draw a game from a distribution on the set of two-player games with a given size. We compute the distribution and the expectation of the number of pure-strategy Nash equilibria of the game conditional on the game having nondecreasing best-response functions. The conditional expected number of pure-strategy Nash equilibria becomes much larger than the unconditional expected number as the size of the game grows.
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JEL classification: C70; C72

Keywords: Number of Nash equilibria; Random game; Strategic complementarity; Increasing difference; Single crossing

1. Introduction

It has been widely recognized among economists that games with strategic complementarities tend to have multiple equilibria. This intuition is probably based on two-player coordination games since they exhibit both strategic complementarities and multiplicity of equilibria.[1] There has been, however, no formal result suggesting any relation between the two properties. In particular, neither implies the other. This paper is the first attempt to provide a basis for this intuition. Namely, we pose the following probability-theoretic question: How likely is it that a game with

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[1] A *two-player coordination game* is a game in which each player's best response to any pure strategy is to play the same pure strategy as the opponent (Young, 1998).

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doi:10.1016/j.geb.2007.10.003

strategic complementarities has multiple equilibria? Following the literature on random games, we randomly draw a game from a distribution on the set of two-player games with a given size. We compute the distribution and the expectation of the number of pure-strategy Nash equilibria (PNEs) of the game conditional on the game having nondecreasing best-response functions. We show that the conditional expected number of PNEs becomes much larger than the unconditional expected number as the size of the game grows.

Our statistical model is the following. A univariate random variable or a distribution on $\mathbb{R}$ is *continuous* if its cumulative distribution function is continuous. We randomly generate a normal-form game with a given size by independently drawing all payoff values from an identical continuous distribution. Then we count the number of PNEs in this game. (Note that, since the distribution of payoff values is continuous, every PNE is almost surely strict.) In this setup, the game has one PNE on average. Powers (1990) shows that the number of PNEs converges in distribution to the Poisson distribution with mean 1 as the strategy sets become large. Analogous results are obtained for symmetric games as well (Stanford, 1996).

This paper analyzes two-player games with nondecreasing best-response functions. Note that Tarski's fixed point theorem implies that every finite game with nondecreasing best-response functions has at least one PNE (Topkis, 1979). We show that the condition that best-response functions are nondecreasing not only guarantees the existence of PNEs, but also substantially increases the average number of PNEs. Namely, in our statistical model with payoff values given by i.i.d. continuous random variables, conditional on that best-response functions be nondecreasing, the expected number of PNEs is asymptotically proportional to $\sqrt{n}$ in a random symmetric $n \times n$ game (Section 3), and $\sqrt{mn/(m+n)}$ in a random asymmetric $m \times n$ game (Section 4).

These results hinge on our statistical model. In particular, best-response functions are *uniformly* distributed on the set of nondecreasing functions in our model. There may be, however, other sensible statistical models. Such models are discussed in Section 5.

Our results suggest that the prediction power of Nash equilibrium is weak especially in games with strategic complementarities. Since traditional equilibrium refinements such as perfection do not eliminate any strict Nash equilibrium, we need a rather strong criterion of equilibrium selection to pin down the most reasonable PNE. In the recent literature on equilibrium selection, much attention has been paid to supermodular games. A game is *supermodular* if a change in a player's payoffs from one strategy to a higher one is nondecreasing in the profile of his opponents' strategies. Every supermodular game has nondecreasing best-response functions, but not vice versa. In the class of supermodular games, global games and perfect foresight dynamics have been shown to be well-behaved equilibrium-selection approaches (Frankel et al., 2003; Oyama et al., 2003; Takahashi, in press).

1.1. Related literature

Other classes of games have been analyzed in the literature on random games. For example, two-player zero-sum games have fewer PNEs than non-zero-sum games. Namely, if player 1's payoff values are given by i.i.d. continuous random variables, and, for each strategy profile, player 2's payoff is given by the negative of player 1's payoff, then almost every $m \times n$ zero-sum game has at most one PNE, and the probability that a random $m \times n$ zero-sum game has a PNE is given by $m!n!/(m+n-1)!$, which converges to 0 as $\min(m,n) \geqslant 2$ and $\max(m,n) \to \infty$ (Goldman, 1957).

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