Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Physica A 355 (2005) 602–618
PHYSICA A
www.elsevier.com/locate/physa

Congestion schemes and Nash equilibrium in complex networks

Juan A. Almendral[a], Luis López[b], Vicent Cholvi[c], Miguel A.F. Sanjuán[a,*]

[a]*Grupo de Dinámica No Lineal y Teoría del Caos, Departamento de Matemáticas y Física Aplicadas y Ciencias de la Naturaleza, Universidad Rey Juan Carlos, Móstoles, 28933 Madrid, Spain*
[b]*Laboratorio de Algoritmia Distribuida y Redes, Departamento de Informática, Estadística y Telemática, Universidad Rey Juan Carlos, Móstoles, 28933 Madrid, Spain*
[c]*Departamento de Lenguajes y Sistemas Informáticos, Universitat Jaume I, 12071 Castellón, Spain*

Received 21 December 2004
Available online 11 May 2005

Abstract

Whenever a common resource is scarce, a set of rules are needed to share it in a fairly way. However, most control schemes assume that users will behave in a cooperative way, without taking care of guaranteeing that they will not act in a selfish manner. Then, a fundamental issue is to evaluate the impact of cheating. From the point of view of game theory, a Nash equilibrium implies that nobody can take advantage by unilaterally deviating from this stable state, even in the presence of selfish users. In this paper we prove that any efficient Nash equilibrium strongly depends on the number of users, if the control scheme policy does not record their previous behavior. Since this is a common pattern in real situations, this implies that the system would be always out of equilibrium. Consequently, this result proves that, in practice, oblivious control schemes must be improved to cope with selfish users.

PACS: 89.75.Fb; 89.65.Gh; 89.70.+c

Keywords: Complex networks; Game theory; Nash equilibrium

*Corresponding author. Tel.: +34 916647443; fax: +34 916647490.
E-mail address: msanjuan@escet.urjc.es (M.A.F. Sanjuán).

doi:10.1016/j.physa.2005.03.023

1. Introduction

The problem of control schemes constitutes a largely studied issue in the past few years [1–7]. Many systems nowadays are based on the principle of sharing a common resource, e.g., a communication link, among different users. Consequently one of the main objectives of such schemes is to establish a number of rules guaranteeing that the common resources are shared in a fair way among users.

However, most of these schemes require users to behave in a cooperative way, so that they respect some "social responsible" rules. Moreover, without forcing end users to adopt a centralized mandated policy controlling their behavior, it is not possible to guarantee that they will not act in a selfish manner. Then, it seems a main issue to evaluate the impact of having users acting this way.

An example that illustrates the above-mentioned scenario is the control scheme used by the TCP/IP protocol, which is currently the dominant protocol in the internet. By using it, users control the injection rate of packets into the communication network by means of a pair of parameters. When users detect that the network is overloaded, which is done by means of some control messages, they decrease their injection rate by a half, thus alleviating the network's load. However, the adherence to this scheme is voluntary in nature, and some users may decide to act in a selfish manner and not to decrease its injection rate. As it has been evaluated by several authors [8,9], this may lead to a congestion collapse that only benefits selfish users. Therefore, it is interesting to know how cooperative users may "fight back" against unsupportive ones.

Another example of congestion can be found in social networks, mainly when they are based on the traditional hierarchical paradigm. Despite the problems of congestion showed by this topology, large companies prefer this hierarchical organization because it is the only way to keep their activities under a strict control. But with the growth of the companies, the number of specialized activities grows also, and it is needed to introduce a non-hierarchical communication to maintain the efficiency. However, nobody knows how to control a non-hierarchical organization. The hope is that some global "self-organizing" order emerges by means of a horizontal protocol [10]. Consequently, it is important to analyze the features which such a successful protocol must have.

Game theory constitutes a good mathematical tool for analyzing the interaction of decision makers with conflicting interests [11,12]. From a game-theoretic perspective, users are considered the *game players* and congestion control schemes establish the *game rules*.

We regard *players* as agents that issue requests for a common resource selfishly (i.e., they are only concerned about their own good). Hence, the utility function of each player, which is the parameter to be maximized, is assumed to be equal to the number of requests that have been served per unit time.

The *rules* of the game are determined by the management policy of the common resource. Here, we consider policies that are *oblivious*, i.e., that do not differentiate between requests belonging to different agents, and that have a limited storage capacity for pending requests. Moreover, requests issued after such a limit is