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# Correlated Nash equilibrium

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**Abstract**

We modify the epistemic conditions for Nash equilibrium only to accommodate Gilboa and Schmeidler's [I. Gilboa, D. Schmeidler, Maxmin expected utility with nonunique prior, J. Math. Econ. 18 (1989) 141–153] maxmin expected utility preferences, and identify the equilibrium concept in $n$-player strategic games that characterizes the modified epistemic conditions. The epistemic characterization supports the equilibrium concept as a minimal generalization of Nash equilibrium, in the sense that it deviates from Nash equilibrium only in terms of players' attitude towards ambiguity. Consequently, comparing it with Nash equilibrium constitutes a ceteris paribus study of the effects of ambiguity on how a game is played. For example, with ambiguity, (beliefs about) action choices are in general correlated.


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## 1. Introduction

Nash equilibrium [30] has been the central solution concept in game theory. Adopting the view that choosing an action in a game is a decision problem under uncertainty, Aumann and Brandenburger [4] provide epistemic conditions for Nash equilibrium; more precisely, they prove that Nash equilibrium characterizes mutual knowledge of rationality, common knowledge of beliefs, and the common prior assumption.

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There is no question that (the epistemic conditions for) Nash equilibrium can be criticized from a number of directions. Two of them, which are relevant to this paper, are as follows. The first criticism is that the notion of rationality is too restrictive, in the sense that players' preferences have to be represented by the expected utility model of Savage [34]. A prominent property of expected utility is what Machina and Schmeidler [27] call *probabilistic sophistication*. Roughly speaking, probabilistic sophistication says that a decision maker behaves as if his beliefs are represented by a probability measure. However, the Ellsberg Paradox [10] and related experimental findings (summarized by Camerer and Weber [7]) demonstrate that when there is ambiguity (about the probability law governing the uncertainty), probabilistic sophistication may be unrealistic; moreover, a decision maker is typically ambiguity averse, which roughly means that he prefers to bet on events with known rather than unknown probabilities. Although the Ellsberg Paradox involves only a single decision maker facing an exogenously specified environment, it suggests that ambiguity should also be common in strategic situations. For instance, it is intuitive that a player may be ignorant about what his opponents would do "off the equilibrium path" (that is, in contingencies that are not supposed to be realized). More generally, it just seems hard for a player to predict (even in a probabilistic sense) what his opponents are going to do, whenever they have multiple best responses.

Second, casual observations suggest that people often, explicitly or implicitly, agree to disagree on what other people are going to do. In almost any public discussion of current affairs, the people involved could easily find at least some disagreement (even when the discussion is over) on how a political or economic episode will unfold. A dramatic real-life story can be found in Greenberg [14, p. 197], who cites Kissinger's [18, p. 802] view on the peace talks between Egypt and Israel following the 1973 war. It appeared that the two countries (at least implicitly) agreed to disagree on what the United States would do if negotiations broke down (which could be interpreted as a contingency off the equilibrium path), and that was crucial to the success of the talks. The phenomenon of agreeing to disagree provides evidence for common knowledge of beliefs. But if beliefs were probabilistic, agreeing to disagree would contradict the common prior assumption (Aumann [2]).

This paper is motivated by the two criticisms mentioned above. A natural way to escape from the criticisms is to allow beliefs to be represented by not necessarily one probability measure, but a set of them. In fact, "multiple priors" is a key feature of many generalized expected utility models. Gilboa and Schmeidler's [13] maxmin expected utility, which will be the focus of this paper, has become a classic example. As its name suggests, the model portrays an ambiguity-averse decision maker who evaluates any act according to its minimum expected utility, where the minimum is taken over the probability measures in his set of priors.

We modify the epistemic conditions for Nash equilibrium only to accommodate maxmin expected utility preferences, and identify the characterization—*correlated Nash equilibrium*—of the modified epistemic conditions in $n$-player strategic games. The result supports correlated Nash equilibrium as a *minimal* generalization of Nash equilibrium, in the sense that it deviates from Nash equilibrium *only* in terms of players' attitude towards ambiguity. Consequently, comparing it with Nash equilibrium constitutes a ceteris paribus study of the effects of ambiguity on how a game is played.

There are some existing generalized Nash equilibrium concepts for (variants of) maxmin expected utility preferences. In terms of coverage and/or foundations, none of them is complete. Some only cover two-player games (e.g., Dow and Werlang [8], Lo [20], Marinacci [28], Ryan [33]); some cover $n$-player games (e.g., Eichberger and Kelsey [9], Groes et al. [15], Klibanoff [17], Lo [21]), but formal epistemic foundations of those concepts are not given; some