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Invited Review

Dynamic programming and board games: A survey

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Abstract

In several of the earliest papers on dynamic programming (DP), reference was made to the possibility that the DP approach might be used to advise players on the optimal strategy for board games such as chess. Since these papers in the 1950s, there have been many attempts to develop such strategies, drawing on ideas from DP and other branches of mathematics. This paper presents a survey of those where a dynamic programming approach has been useful, or where such a formulation of the problem will allow further insight into the optimal mode of play.

Keywords: Dynamic programming; Games; Leisure

1. Introduction

In the book, ''Fights, Games and Debates'' [34], the three areas of ''conflict'' are distinguished: Fights—where you harm your opponent; Debates—where you convince your opponent; Games—where you outwit your opponent. This paper considers the third, looking at games played on a board, possibly with one or more dice or with playing cards or with pencil and paper. Many examples are also available in electronic form. The theme of games means that there is no physical contact (as in a fight), or verbal skill (as in a debate), but logical skill, in outwitting the opponent or in successfully solving a puzzle set by ''nature''.

Such games have been studied, more or less seriously, for many years, and there is a wide-ranging literature about them, drawing on ideas from pure mathematics, logic and computer science. In some cases, such as computer chess programs, and the exact method of Little et al. [25] for the travelling salesman problem, study of a game or puzzle has contributed to a broader research area. This paper concentrates on the

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approach of dynamic programming (DP), and the contribution (direct or indirect) that DP has made to the investigation of the logical skills needed to solve a puzzle or defeat an opponent in a board game.

2. A structure for DP studies

There are numerous ways in which one might study the use of dynamic programming in "board" games. In what follows, the classification "board" is a generous one. It is intended as a way of distinguishing the games from sports and other physical games. In the paper, we consider a range of situations where the "game" is played by one person against another, or by one person against a "position" that has been set up by chance. Success is based on intellect, mental agility and logic, rather than fitness.

Throughout the paper, we use the traditional terms which are always used in dynamic programming formulations, including: stage, state, policy, decision, recurrence relation; and the expression: curse of dimensionality. Formal definitions will be found in most references to dynamic programming.

Informally, in the general context of board games, the stage is a measure of how many decisions or "moves" remain for a player. In some of the games considered, there are potentially an infinite number of stages remaining, and in such circumstances, the functional recurrence of DP is independent of the stage. The state throughout the paper is the position of the playing pieces (pieces includes playing cards or similar) on the board in front of the player. The decision is what to do next; this will mean an action (usually a move) which changes the state of the board. The policy and recurrence relation imply a value which may be associated with the state and stage. The dimensionality depends on the game, and on how one chooses to describe the state.

The paper is structured to examine games classified in according to the number of players (one or two) and whether or not there is randomness. Sections 3 and 4 consider games played by one person. Section 3 focuses on deterministic games, Section 4 on those where a move has a stochastic element. Then the paper examines two player games, again with a focus on deterministic games (Section 5) and stochastic games (Section 6). Each section is reasonably self-contained.

3. One player, deterministic games

We start with the simplest of all board games. There is one player, who has complete knowledge of the state of the board, and about the consequence of any of his (or her) decisions. Each move produces a known change in the state. (From now on the players will be considered as male.)

Games in this category include:

- Solving the Towers of Hanoi from a given position.
- Unscrambling a Rubik's cube.
- Rearranging the 15-puzzle.
- Solving Freecell (game included in Windows).
- Solving some forms of the family of card games, Patience.
- Solitaire (the board game) problems.

The first common feature of these games is that the player is trying to achieve a given target, and wants to make moves which will achieve that goal. In the towers of Hanoi, the aim is to pile the discs into a specified order. Given a scrambled Rubik's cube, one wants to be able to restore it correctly. With a scrambled 15-puzzle, one wants to achieve the sequential pattern. In the card games, the aim is to collect the cards into particular "stacks".