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A note on the stability of a Cournot–Nash equilibrium: the multiproduct case with adaptive expectations

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Abstract

Recently, Zhang and Zhang [Zhang, A., Zhang, Y. (1996). Stability of a Cournot–Nash equilibrium: the multi-product case. Journal of Mathematical Economics, 26, 441–462.] have presented sufficient and necessary conditions for the asymptotical stability of dynamic multiproduct oligopolies with Cournot expectations. First, we show via a counterexample that the necessary conditions given in that paper do not necessarily hold, and then their sufficient conditions are extended to adaptive expectations. Simple sufficient conditions are finally given by using special matrix norms. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

The stability of dynamic multiproduct oligopolies was recently examined by Zhang and Zhang (1996), who have presented sufficient conditions for the

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asymptotical stability of Nash–Cournot equilibria. These results are the nonlinear extensions of the linear models given in (Okuguchi and Szidarovszky, 1990), and are also analogous to the nonlinear models given in the same book. Zhang and Zhang have considered only the case of Cournot expectations, and their necessary conditions were based on a result the proof of which has an error. In this paper the stability of dynamic multiproduct oligopolies will be investigated under adaptive expectations. The adaptive scheme of this paper is the same as the one given earlier in (Okuguchi and Szidarovszky, 1990). Our analysis will be based on the following theorem.

Proposition 1.1: *Let $\underline{z}^*$ be an interior equilibrium of the discrete dynamic system $\underline{z}_{t+1} = \underline{T}(\underline{z}_t)$, where $\underline{T}: D \mapsto D$ is continuously differentiable with $D \subseteq \mathbb{R}^N$ being an arbitrary set. Let $\underline{T}'$ denote the Jacobian of $\underline{T}$.*

1. *If all eigenvalues of $\underline{T}'(\underline{z}^*)$ are inside the unit circle, then $\underline{z}^*$ is locally asymptotically stable*;
2. *If at least one eigenvalue of $\underline{T}'(\underline{z}^*)$ is outside the unit circle, then $\underline{z}^*$ is unstable*.

Part 1 is well known from systems theory, since if all eigenvalues of $\underline{T}'(\underline{z}^*)$ are inside the unit circle, then there is a matrix norm such that $\|\underline{T}'(\underline{z}^*)\| < 1$ (see for example, Ortega and Rheinboldt, 1970). Part 2 is known from the stable and unstable manifold theorem (see for example, Katok and Hasselblatt, 1997), and recently an elementary proof for this result has been given by Li and Szidarovszky (1999). We mention that part 2 can be reformulated as a necessary stability condition as follows:

2^*. *If $\underline{z}^*$ is asymptotically stable, then for all eigenvalues λ_i of $\underline{T}'(\underline{z}^*)$, $|\lambda_i| \le 1$.*

We also mention that part 2 cannot be stated via matrix norms, since if $\underline{z}^*$ is asymptotically stable, then it is possible that for all matrix norms, $\|\underline{T}'(\underline{z}^*)\| > 1$. Such an example is given by the system with

$$\underline{T}(\underline{z}) = \begin{pmatrix} xe^{-x^2} + ye^{-y^2} \\ ye^{-y^2} \end{pmatrix}, \tag{1.1}$$

where the unique equilibrium is $x^* = y^* = 0$ and

$$\underline{T}'(\underline{z}^*) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

An elementary analysis shows that $\underline{z}^* = \underline{0}$ is asymptotically stable, the only eigenvalue of $\underline{T}'(\underline{z}^*)$ is 1 with multiplicity 2, therefore condition 2^* is satisfied. However, it can be verified that for all matrix norms, $\|\underline{T}'(\underline{z}^*)\| > 1$ (also see the