Mathematical Social Sciences 41 (2001) 103–110

mathematical social sciences

www.elsevier.nl/locate/econbase

An algorithm for verifying double implementability in Nash and strong Nash equilibria

Sang-Chul Suh*

Department of Economics, University of Windsor, Windsor, Ontario, Canada N9B 3P4

Received 1 January 1999; received in revised form 1 November 1999; accepted 1 November 1999

Abstract

Suh (Suh, S., 1997. Double implementation in Nash and strong Nash. Social Choice and Welfare 14, 439 − 447.) considered a decision making problem where there are a set of alternatives and a finite number of agents with preferences defined over the set of alternatives, and provided a necessary and sufficient condition for double implementation in Nash and strong Nash equilibria. One problem we encounter in Suh's paper is that it is difficult to apply the condition directly to a given (social choice) correspondence. Here we provide an algorithm which helps us to verify whether a correspondence satisfies the condition or not.

Keywords: Double implementation; Nash and strong Nash equilibria; Algorithm

JEL classification: C72; D78

1. Introduction

Consider an abstract model of a decision making problem where there are a set of alternatives and a finite number of agents with preferences defined over the set. A (*social choice*) *correspondence* assigns each preference profile a set of alternatives. To achieve the goal of selecting alternatives which are recommended by the correspondence, information about agents' preferences is needed. But the information may not be publicly known and agents may behave strategically. We are interested in investigating the possibility of constructing a decentralized institution (or mechanism) to achieve the goal in such a situation. A *mechanism* consists of a set of strategies for each agent and a

*Corresponding author. Tel.: +1-519-253-3000, ext. 2384; fax: +1-519-973-7096.
E-mail address: scsuh@uwindsor.ca (S.-C. Suh).

0165-4896/01/$ – see front matter
PII: S0165-4896(99)00057-8

function which assigns each strategy profile an alternative. Suppose that an equilibrium concept, Z, describes agents' strategic behaviors. If the set of Z equilibrium outcomes of the game given by the mechanism coincides with the set of alternatives selected by the correspondence for all possible preference profiles, then we say that the mechanism *implements* the correspondence in Z equilibrium.

Maskin (1977) provided a characterization result for Nash implementation and Dutta and Sen (1991a) provided a full characterization result for strong Nash implementation.[1] While the Nash equilibrium concept applies to environments where agents cannot form coalitions, hence agents can make only unilateral deviations, the strong Nash equilibrium concept applies to environments where any possible group of agents can form coalitions. Suh (1996b) provided a full characterization result for the equilibrium concept which applies to environments where general coalition formation is possible. One important assumption implicit in those papers mentioned is that the planner has information about who can form coalitions with whom.

What correspondences are implementable if the planner does not have the information about coalition formation possibilities among agents? This question in effect asks which correspondences are (doubly) implementable both in Nash and strong Nash equilibria. Suh (1997) provided an answer to this question: a necessary and sufficient condition for double implementation is provided.[2] Although the condition completely characterizes doubly implementable correspondences, it is difficult to apply the condition directly to a correspondence in question.

We provide an algorithm which allows us to verify whether a correspondence satisfies the condition for double implementation. Sjöström (1991) gave an algorithm for checking necessary and sufficient conditions for Nash implementation. We extend his idea in constructing the algorithm for double implementation.

We organize the remainder of the paper as follows. After introducing notation and definitions in Section 2. We provide an algorithm which allows us to verify whether a rule satisfy the condition for double implementation in Section 3.

2. Notation and definitions

Let A be an arbitrary set of alternatives. Let $N = \{1, \ldots, n\}$ be a finite set of agents. Each *agent* $i \in N$ has preferences R_i, a binary relation on A which is complete, transitive and reflexive. Let P_i be the strict preference relation and I_i be the indifference relation. Let $\Re_i$ be the set of agent i's admissible preferences and $\Re = \Re_1 \times \cdots \times \Re_n$. A *preference profile* is a list $R = (R_1, \ldots, R_n) \in \Re$. A (*social choice*) *correspondence* is a mapping F which associates with each preference profile $R \in \Re$ a non-empty subset of A.

[1] For Nash implementation, refer to Williams (1986), Saijo (1988), Danilov (1992), Moore and Repullo (1990) and Dutta and Sen (1991b), and for strong Nash implementation, refer to Maskin (1979), Moulin and Peleg (1982) and Suh (1996a).

[2] For the issue of constructing a 'desirable' mechanism which doubly implements a certain correspondence, refer to Suh (1995), Peleg (1996a,b), Tian (1996a,b), Yoshihara (1999) and Shin and Suh (1997).