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Decision Support

Exact and heuristic algorithms for the design of hub networks with multiple lines

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ABSTRACT

In this paper we study a hub location problem in which the hubs to be located must form a set of interconnecting lines. The objective is to minimize the total weighted travel time between all pairs of nodes while taking into account a budget constraint on the total set-up cost of the hub network. A mathematical programming formulation, a Benders-branch-and-cut algorithm and several heuristic algorithms, based on variable neighborhood descent, greedy randomized adaptive search, and adaptive large neighborhood search, are presented and compared to solve the problem. Numerical results on two sets of benchmark instances with up to 70 nodes and three lines confirm the efficiency of the proposed solution algorithms.

1. Introduction

Hub-and-spoke architectures are often used in the design of large-scale networks such as those found in passenger and freight transportation, postal services, telecommunications, and rapid transit systems. In these networks, commodities from different origins are sent to intermediate facilities, known as hubs, which are responsible for the aggregation and distribution of the flows to multiple destinations. This allows the connection of a large number of origin/destination (O/D) nodes with a small number of arcs, reducing the infrastructure and operational cost (O'Kelly & Miller, 1994). Another important advantage of hub-and-spoke networks is that hub facilities can be connected with highly efficient pathways, enabling economies of scale to be achieved on the transportation cost (or travel time) between hubs. *Hub location problems* (HLPs) consider the design of hub networks by selecting a set of nodes to locate hubs, activating a set of links, and routing commodities through the network while optimizing a cost-based (or service-based) objective function. We refer the reader to Alumur and Kara (2008), Campbell and O'Kelly (2012), Farahani, Hekmatfar, Arabani, and Nikbakhsh (2013), and Contreras, Laporte, Saldanha da Gama, and Nickel (2015) for recent surveys on hub location.

Given the inherent difficulty of HLPs, most of the fundamental HLPs consider a fully interconnected hub-level network to simplify the network design decisions. However, it is known that this can be an oversimplification in applications where there is a considerable set-up cost associated with the inter-hub links (see O'Kelly & Miller, 1994). Several HLPs considering incomplete hub-level networks have thus been studied. These problems can be seen from a hub arc location perspective (see Campbell, Ernst, & Krishnamoorthy, 2005a; 2005b; Contreras & Fernández, 2014), in which the location of a set of hub arcs and their associated hub nodes is considered. Motivated by specific applications, some of these models require the hub-level network to have a particular topological structure, such as cycles (Contreras, Tanash, & Vidyarthi, submitted for publication; Lee, Ro, & Tcha, 1993), stars (Labbé & Yaman, 2008), trees (Contreras, Fernández, & Marín, 2009; Contreras, Fernández, & Marín, 2010; Martins de Sá, de Camargo, & de Miranda, 2013), or lines (Martins de Sá, Contreras, Cordeau, de Camargo, & de Miranda, in press). Some other models do not even require the hub arcs to define a single connected component (Campbell et al., 2005a; Contreras & Fernández, 2014).

The *hub line location problem* (HLLP), introduced in Martins de Sá et al. (in press), consists of designing a hub network in which p hubs are located and connected by means of a single line. Contrary to most p-hub median models considering a cost-based objective, the HLLP uses a service-based objective that aims at minimizing the total weighted travel time between O/D pairs. It considers that a high-speed mode of transportation is available on the hub arcs and thus, their travel speed is faster than on the other links of the network. The total travel time when using the hub line takes into account

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the access and exit times that may exist when using the hub line due to a change in mode of transportation or to waiting because of frequency or congestion related issues. The trade-off between the benefit of using a high-speed mode of transportation to efficiently travel and the added time for interacting with the hub line make the routing decisions more involved. Demand flow must be routed via either a path using a segment of the hub line or with a direct connection between origin and destination, depending on whichever route provides the smallest travel time. Potential applications of hub line networks arise in the location of extensive facilities and in public transportation planning, in particular in the design of rapid transit systems such as a subway, tram or light rail lines. Hub facilities then correspond to stations where a change of mode of transportation is possible. Non-hub nodes may represent districts, bus stops or taxi stations. Users will employ the hub line if there is a reduction in their travel time or they will keep using the shortest route on the existing network. For additional details on the HLLP the reader is referred to Martins de Sá et al. (in press).

One of the limiting aspects of the HLLP is that it is only applicable to situations where the hub network will have exactly one line with a predetermined number of hubs. In this paper we generalize the HLLP to the case in which the hub network is composed by more than one line. In particular, we introduce the *q-line hub location problem* (q-HLLP) which consists of locating a set of q lines that minimize the total travel time between O/D pairs, while satisfying a budget constraint on the total setup cost of the network associated with the location of hub nodes and hub arcs. As in the HLLP, we assume that O/D nodes can be assigned to more than one hub node, i.e. a multiple allocation pattern. However, instead of considering a predetermined number of hub nodes in a line, the q-HLLP considers as part of the decision process the determination of the number of hubs contained in each line, while respecting lower and upper limits on this number and the budget constraint for the total setup cost. In order to properly model the total travel time when using more than one hub line, a waiting time to transfer between lines needs to be taken into account. For instance, when transferring lines at a subway station, the average time spent walking between gates and waiting for the next subway train to pass, which depend on the size of the station and train frequency, could be significant. These transfer times might not compensate the reduction of travel time from using the subway, especially if transferring more than once, and thus users may continue traveling as before. These times make the q-HLLP more general and thus, more challenging to formulate and solve.

In the standard version of the q-HLLP, the hub-level network is not required to define a single connected component. An example of a potential application in which this is not a requirement arises in bus transit systems, where sets of express bus lines may be disconnected if they run in very different areas within a metropolitan area. However, in other applications such as in subway systems, the metro lines are always connected. Therefore, we also consider a variant of the q-HLLP in which it is assumed that the hub-level network must be connected.

Fig. 1 illustrates two different 2-line hub networks that have the same topological structure, but actually represent different systems when transfer times are taken into account. For example, if transfer times are strictly positive, the travel time from hub node 1 to hub node 3 in hub network 1 is smaller than in hub network 2, since the latter implies a transfer from line 2 to line 1 at hub node 2.

To the best of our knowledge, the design of transportation networks considering multiple lines has not been previously addressed from a hub location perspective. However, it has been considered in the context of extensive facility location and rapid transit systems design. An extensive facility is a facility considered to be too large for being represented as a single point when comparing its scale with its interaction environment (Mesa & Boffey, 1996). A review of extensive facility location problems can be found in Mesa and Boffey (1996), who also cover multiple path location problems.

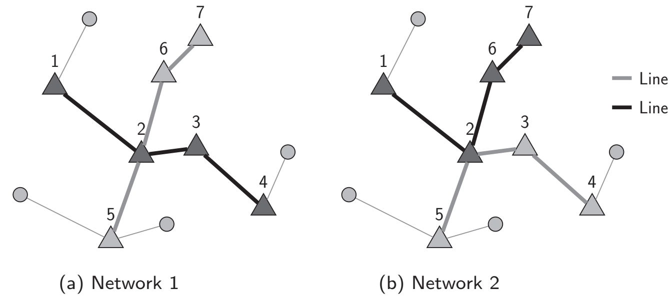

Fig. 1. Illustration of a 2-line hub network.

In the context of the design of rapid transit systems, Bruno, Ghiani, and Improta (1998) address the design of a multi-modal rapid transit line. The problem consists of designing a bi-modal pedestrian–public network and considers a bi-objective function composed of the minimization of the construction costs and the minimization of the total weighted travel cost. It is assumed that the total travel cost associated with each O/D pair is equal to the minimum between the shortest path covered by means of the private system and the shortest path in the pedestrian–public network, which account for the total travel cost to transit between two nodes of the network and the costs associated to the waiting times to boarding and alighting in a station of the rapid transit line. García et al. (2006) address the design of a rapid transit system composed of multiple lines that maximizes the total weighted trip coverage by the system, where the extremities of each line are given. It is assumed that the total cost to satisfy the demand of each O/D pair by the system is equal to the sum of the total travel cost to move in the transit vehicle and the costs associated with transferring from one line to another. In this case, a demand is covered by the system if the total travel cost is lower than the travel cost associated to a competing private system.

Another line of research in rapid transit system design focuses on maximizing the population coverage provided by the network, an objective that differs from the one considered in the q-HLLP. For example, Laporte, Marín, Mesa, and Ortega (2007) present a mathematical formulation for the problem of designing a multiple-line network that maximizes the trip coverage in competition with a private mode taking into account a budget constraint. Marín (2007) presents an extension to the problem proposed by Laporte et al. (2007), where stations are not determined a priori and the number of lines is free but has an upper bound. The problem aims to maximize the public coverage (the main objective) and to minimize the routing cost. Marín and Jaramillo (2008) propose a long term planning model that aims to determine a network capacity expansion plan, i.e., to install additional lines or stations. Marín and Jaramillo (2009) present a Benders decomposition algorithm to solve the urban rapid transit design proposed by Marín (2007).

As noted by Martins de Sá et al. (in press), the design of hub line networks is a very challenging optimization problem, even for the case of a single line. The best Benders decomposition variant presented by the authors for the HLLP can consistently solve to optimality instances with up to 50 nodes and for some particular configurations of the parameters of the HLLP, it can solve instances with up to 100 nodes in one day of CPU time. In this paper, we present exact and heuristic algorithms for designing hub line networks with multiple lines. In particular, we present a mixed-integer programming (MIP) formulation for the q-HLLP which is used in a Benders decomposition algorithm to obtain optimal solutions for small instances and to provide bounds for larger instances. We also develop three different metaheuristics to provide feasible solutions to large instances: *(i)* a variable neighborhood descent (VND), *(ii)* a greedy randomized adaptive search procedure (GRASP) and, *(iii)* an adaptive large neighborhood search (ALNS). In order to evaluate the efficiency