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Discrete optimization

Dynamic programming algorithms for the bi-objective integer knapsack problem

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ABSTRACT

This paper presents two new dynamic programming (DP) algorithms to find the exact Pareto frontier for the bi-objective integer knapsack problem. First, a property of the traditional DP algorithm for the multi-objective integer knapsack problem is identified. The first algorithm is developed by directly using the property. The second algorithm is a hybrid DP approach using the concept of the bound sets. The property is used in conjunction with the bound sets. Next, the numerical experiments showed that a promising partial solution can be sometimes discarded if the solutions of the linear relaxation for the subproblem associated with the partial solution are directly used to estimate an upper bound set. It means that the upper bound set is underestimated. Then, an extended upper bound set is proposed on the basis of the set of linear relaxation solutions. The efficiency of the hybrid algorithm is improved by tightening the proposed upper bound set. The numerical results obtained from different types of bi-objective instances show the effectiveness of the proposed approach.

1. Introduction

The multi-objective integer knapsack problem (MOIKP) is one of the most widely studied multi-objective combinatorial optimization problems where every solution is evaluated according to two or more objectives (Ehrgott, 2005). Most real-life decision problems have to deal with several conflicting criteria which must be optimized simultaneously. MOIKPs can be encountered in a wide range of applications such as capital budgeting (Klamroth & Wiecek, 2000; Kwak, Shi, Lee, & Lee, 1996; Rosenblatt & Sinuany-Stern, 1989), selection of different investment projects (Alanne, 2004; Bas, 2011; Teng & Tzeng, 1996), or relocation issues arising in nature conservation (Kostreva, Ogryczak, & Tonkyn, 1999), or pollution issues for remediation planning (Jenkins, 2002).

A MOIKP can be defined as follows. Consider a knapsack capacity, W, and a set of n items where each item j $(j = 1, \dots, n)$ is associated with a weight w_j and r profit values, c_j^k, one *per* objective k $(k = 1, \dots, r)$; each item j has a number of copies (only bounded by $\lfloor W/w_j \rfloor$) available. The problem consists of determining the number of copies for each item such that the overall weight does not exceed W and the total profits are maximized in a Pareto sense. The MOIKP can also be stated as a multi-objective integer linear programming model as follows:

$$\begin{aligned} \text{vmax} \quad & \left(\sum_{j=1}^{n} c_j^1 x_j, \dots, \sum_{j=1}^{n} c_j^r x_j \right) \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j x_j \leqslant W, \\ & x_j \geqslant 0, \quad \text{integer}, \quad j = 1, \dots, n, \end{aligned} \qquad \text{(MOIKP)}$$

where x_j are decision variables indicating the number of copies for the jth item placed in the knapsack. It is assumed that $W,\ w_j\ (\leqslant W)$ and c_j^k $(j = 1, \dots, n,\ k = 1, \dots, r)$ are positive integers.

Let $f_k(\mathbf{x}) = \sum_{j=1}^{n} c_j^k x_j,\ k = 1, \dots, r$. Solving the MOIKP is interpreted here as generating its efficient set $\mathcal{X}_E$ in the decision space $\mathcal{X} \subseteq \mathbb{R}^n$ and the corresponding image $\mathcal{Y}_N = f(\mathcal{X}_E)$ in the objective space $\mathbb{R}^r$, called the *non-dominated set* or the *Pareto frontier* (PF).

The dominance relations are defined based on the componentwise ordering of $\mathbb{R}^r$. For $\mathbf{y}^1, \mathbf{y}^2 \in \mathbb{R}^r$,

$$\begin{aligned} \mathbf{y}^1 \geqslant \mathbf{y}^2 \quad & \Longleftrightarrow \quad y_k^1 \geqslant y_k^2, \quad k = 1, \dots, r \\ \mathbf{y}^1 \geqslant \mathbf{y}^2 \quad & \Longleftrightarrow \quad y_k^1 \geqslant y_k^2, \quad k = 1, \dots, r \quad \text{and} \quad \mathbf{y}^1 \neq \mathbf{y}^2 \\ \mathbf{y}^1 > \mathbf{y}^2 \quad & \Longleftrightarrow \quad y_k^1 > y_k^2, \quad k = 1, \dots, r. \end{aligned}$$

The relations $\leqq$, $\leqslant$ and $<$ are defined accordingly. $f(\bar{\mathbf{x}}) \in \mathbb{R}^r$ is *dominated* by $f(\mathbf{x}) \in \mathbb{R}^r$ if $f(\mathbf{x}) \geqslant f(\bar{\mathbf{x}})$.

$$\mathcal{X}_E = \{\mathbf{x} \in \mathcal{X} : \text{ there exists no } \bar{\mathbf{x}} \in \mathcal{X} \text{ with } f(\bar{\mathbf{x}}) \geqslant f(\mathbf{x})\}$$

In a multi-objective combinatorial optimization context, the computational effort for identifying the PF is large because the problem of determining whether a given objective vector

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(outcome) is dominated or not is $\mathcal{NP}$-hard (Serafini, 1986) even though the underlying single objective optimization problem can be solved in pseudo-polynomial time. For a given number of objectives, a multi-objective optimization with 0-1 decision variables is much easier to handle than the problem with general integer variables because the problem structure of the former is simpler and combinations of the decision variables for the former are fewer. Therefore, algorithms for solving the 0-1 multi-objective knapsack problem (MOKP) are relatively abundant when compared with algorithms especially designed for the MOIKP.

The 0-1 MOKP as well as the MOIKP fall into the category of multi-objective integer problems. Thus, the resolution of the MOIKP can benefit from the algorithmic developments of both the 0-1 MOKP and the multi-objective integer problem. The literature in the field includes exact and approximate approaches. The former are targeted to find the exact PF while the latter aim at approximating the PF. Exact approaches include dynamic programming (DP) algorithms (Bazgan, Hugot, & Vanderpooten, 2009a; Captivo, Clímaco, Figueira, Martins, & Santos, 2003; Delort & Spanjaard, 2010; Figueira, Tavares, & Wiecek, 2010; Figueira, Paquete, Simões, & Vanderpooten, 2013; Rong & Figueira, 2013; Rong, Figueira, & Pato, 2011; Villarreal & Karwan, 1981), branch-and-bound (BB) algorithms (Florios, Mavrotas, & Diakoulaki, 2010; Mavrotas & Diakoulaki, 2005; Mavrotos, Figueira, & Antoniadis, 2011; Ponte, Paquete, & Figueira, 2012; Visée, Teghem, Pirlot, & Ulungu, 1998), and other exact methods as for example the following ones: exact algorithms that exploit the development for multi-objective linear programs (Jolai, Rezaee, Rabbani, Razmi, & Fattahi, 2007), greedy optimal algorithms (Gorski, Paquete, & Pedrosa, 2012) for a special class of multi-dimensional knapsack problems with binary weights, exact algorithms based on a partition of the search space (Dhaenens, Lemestre, & Talbi, 2010) and algorithms (Laumanns, Thiele, & Zitzler, 2006; Mavrotas & Florios, 2013; Özlen & Azizoğlu, 2009; Przybylski, Gandibleux, & Ehrgott, 2010; Sylva & Crema, 2004, 2007; Zhang & Reimann, 2014) resorting to the scalarization techniques (Ehrgott, 2005) such as weighted-sum method and ε-constraint method as well as a general approach for preserving the lexicographic order of efficient solutions in the decision space (Schweigert & Neumayer, 1997). Approximate approaches include polynomial approximation schemes (Bazgan, Hugot, & Vanderpooten, 2009b), tailored heuristics (Zhang & Ong, 2004; Mavrotas, Figueira, & Florios, 2009; Köksalan & Lokman, 2009), and metaheuristics (Alves & Almeida, 2007; Aytg & Sayin, 2009; Ben Abdelaziz, Kirchem, & Chaouadi, 1999; Gandibleux & Fréville, 2000; Gomes da Silva, Clímaco, & Figueira, 2006, 2004; Gomes da Silva, R Figueira, & Clímaco, 2007; Jaszkiewicz, 2004). The interested readers can also be referred to several comprehensive surveys (Ehrgott & Gandibleux, 2000; Jones, Mirrazav, & Tamiz, 2002; Ulungu & Teghem, 1994) for both exact and approximate approaches.

Research on multi-objective integer problems dates back to more than three decades ago when Villarreal and Karwan (1981) proposed both basic and hybrid DP approaches. These authors also proposed for the first time the concept of bound sets for the hybrid approach. A bound set consists of a set of bounds, which helps in discarding dominated outcomes in multi-objective combinatorial optimization problems. Very recently, an operational way to compute bound sets has been devised based on the convex hull of the image of solutions in the objective space (Ehrgott & Gandibleux, 2007; Sourd & Spanjaard, 2008). For the 0-1 bi-objective knapsack problem, Delort and Spanjaard (2010) and Ponte et al. (2012) applied the concept of bound sets to develop a DP algorithm and a BB algorithm respectively. Similarly, Figueira et al. (2013) applied the concept of bound sets to improve the solution efficiency for DP algorithms.

However, for the MOIKPthe upper bound set was only defined conceptually in Villarreal and Karwan (1981) and there was no efficient operational way to compute the upper bound set. Overall, the algorithm development for the MOIKP advances much more slowly when compared with the 0-1 MOKP. In Klamroth and Wiecek (2000), conceptually DP network approaches for different variants of MOIKP were proposed. In Figueira et al. (2010), an implementation of the generic labeling algorithm for the MOIKP and several network models were considered. Very recently, several reduction techniques were introduced in Rong and Figueira (2013) for improving the solution efficiency for the bi-objective integer knapsack problem.

The current research aims at generating the exact PF for the bi-objective integer knapsack problem (BOIKP) using bound sets in the DP algorithm context. Different from Delort and Spanjaard (2010), the bound sets are applied with the multi-objective DP algorithm for solving the original BOIKP directly instead of using the single-objective DP algorithm for solving a sequence of weighted-sum problems in a two-phase framework. In addition, the way of designing bound sets is different. Delort and Spanjaard (2010) did not directly use the set of feasible solutions (the solution of the problem refers to the outcome in the objective space throughout the paper) as a lower bound set considering the non-convexity of the PF. The authors extended the set of feasible solutions to increase the threshold of discarding partial solutions so that promising partial solutions can be kept. The current study directly uses the set of feasible solutions as a lower bound set and guarantees that the promising partial solutions can be kept by extending the set of linear relaxations (upper bound set).

The contributions of the paper can be summarized as follows. First, the numerical experiments showed that for a given partial solution associated with a non-dominated outcome of the problem (promising partial solution), the bound set estimated by the set of linear relaxation solutions of the subproblem cannot always provide a proper upper bound set to guarantee that it is kept during the solution process. Then an extended upper bound set is proposed on the basis of the set of linear relaxation solutions. Second, a property of the traditional DP algorithm for the MOIKP is newly identified in the current research. The first new DP algorithm is developed by directly using the property. The main difference between the new and the traditional DP algorithms (the best algorithm in Figueira et al. (2010)) is that the new algorithm can reduce significantly the number of nodes to aggregate at the last time-consuming aggregating stage. Usually, for the multi-objective DP algorithm, the complete non-dominated set for the problem cannot be obtained at a single node. In stead, it needs to be obtained by a union operation of the outcomes for the nodes at the last stage of the DP process (Figueira et al., 2010). The second hybrid algorithm is developed by using the property in conjunction with bound sets. The property is used to reduce the number of the recorded nodes during the DP process and the above proposed bound set is tightened by controlling the extent to which the linear relaxation solutions are extended.

The paper is organized as follows. Section 2 introduces a property of the DP algorithm for the MOIKP. Section 3 discusses the bound sets for the BOIKP. Section 4 describes the DP algorithm using the bound sets. Section 5 reports the numerical results for the bi-objective instances. A comparison with different benchmark algorithms is provided.

2. A property of the basic sequential DP algorithm

DP (Bellman, 1957) is an optimization approach that decomposes a complex problem into a sequence of simpler subproblems. It can be considered as a recursive process, which interprets an optimization problem as a multi-stage decision process. Associated with each stage of the optimization problem are the states of the process. The