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European Journal of Operational Research 174 (2006) 1220–1246
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

O.R. Applications

Solving a gas-lift optimization problem by dynamic programming

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Received 17 September 2003; accepted 7 February 2005
Available online 6 June 2005

Abstract

Gas lift is a costly, however indispensable means to recover oil from high-depth reservoirs that entails solving the gas-lift optimization problem, GOP, often in response to variations in the dynamics of the reservoir and economic oscillations. GOP can be cast as a mixed integer nonlinear programming problem whose integer variables decide which oil wells should produce, while the continuous variables allocate the gas-compressing capacity to the active ones. This paper extends the GOP formulation to encompass uncertainties in the oil outflow and precedence constraints imposed on the activation of wells. Recursive solutions are suggested for instances devoid of precedence constraints, as well as instances arising from precedence constraints induced by forests and general acyclic graphs. For the first two classes, pseudo-polynomial algorithms are developed to solve a discretized version of GOP, while the most general version is shown to be NP-Hard in the strong sense. Numerical experiments provide evidence that the approximate algorithms obtained by solving the recurrences produce near-optimal solutions. Further, the family of cover inequalities of the knapsack problem is extended to the gas-lift optimization problem.

Keywords: Dynamic programming; Production; Oil industry; Gas-lift optimization; Polyhedra

1. Introduction

To counter the mounting pressure from economic markets and the ever-increasing demand for nonrenewable resources, the oil industry is investing in the development of improved, more efficient technologies in all of its sectors. In particular, the gas-lift operation of oil fields is one of many production processes whose performance can be improved. Because the internal pressure in high-depth or depleted reservoirs

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2005.03.004

can force the flow of only a fraction of its oil to the surface, the use of artificial means becomes imperative to lift the oil, especially so for deep reservoirs found off-shore. Two examples of artificial lifting are submerged pumps and continuous injection of gas. Although the former can in principle recover most of the oil, its operating costs are excessively high for today's oil prices, not mentioning the potential of an unfavorable energy trade and other technical hindrances. The gas-lift technique, on the other hand, harnesses the reservoir's gas by injecting natural gas into the production tubing so as to reduce the weight of the oil column, thereby elevating the mix of oil, gas, and water to the surface. Fig. 1 depicts a small cluster of wells operated by a continuous gas-injection system, consisting of an oil reservoir, a chamber for compressed gas, and a pool of electric compressors. The issues and problems related to the gas-lift production of an oil field abound, ranging from the design of the gas-compressing unit and the gas pipeline, through the modeling of the production flow as a function of the gas injection, to the allocation of compressed gas among wells. Despite recent technological advances, the operating processes can be further improved in a number of ways, especially in the development of more accurate models and the design of algorithmic solutions to the problems thereof, all with the aim of increasing performance and the degree of automation.

Herein, the focus is on the allocation of compressed gas to the wells of a cluster, taking into account issues that play a part in the performance of the process, such as the limits on gas injection, the nonlinear and uncertain nature of the outflow [18], and logic constraints on the activation of the wells. Roughly stated, the problem amounts to first deciding which wells will produce and second, determining the rate of gas-injection for the active ones. This problem, hereafter referred to as the gas-lift optimization problem, GOP, is instantiated and solved continuously over time, often after the models of well outflow are recalibrated and as economic factors oscillate. It consists of a mixed-integer nonlinear optimization problem, whose discrete variables indicate which wells are active and which are not, whose continuous variables gauge the rate of gas injected into the active wells, and whose constraints spell out the physical constraints.

The problem of concern has been the object of investigation in the literature. Grothey and McKinnon [6] formulated the problem as a mixed-integer nonlinear program (MINLP) and analyzed the performance of a specially-tailored branch-and-bound algorithm, a Benders' decomposition approach whose sub-problems are MINLPs, and also a Lagrangian relaxation procedure. The results reported from computational experiments revealed that the approximation approaches could find near-optimal solutions relatively quickly, while the branch-and-bound algorithm failed at the large instances.

Wang et al. [19] tackled a similar problem, in which they converted the MINLP into a mixed-integer linear programming problem (MILP) by piecewise-linearizing the nonlinear constraints and then applying meta-heuristics. The same authors [20] later augmented the problem formulation to account for the flow interactions among wells, but this time the discrete decisions regarding the activation of wells were treated

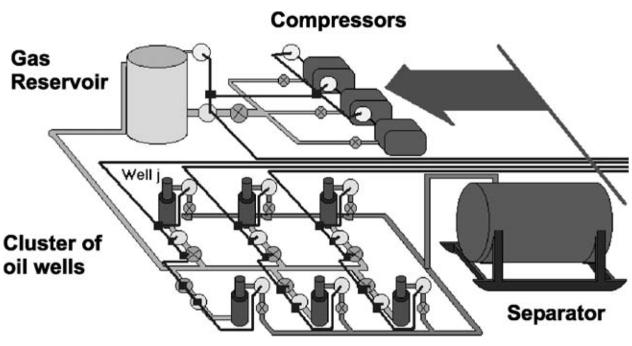

Fig. 1. A cluster of oil wells.