Available online at www.sciencedirect.com
SCIENCE DIRECT®
European Journal of Operational Research 169 (2006) 667–676
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Interfaces with Other Disciplines

A dynamic programming approach to price installment options

Hatem Ben-Ameur [a], Michèle Breton [a,*], Pascal François [b]

[a] Centre for Research on e-finance, GERAD and HEC, 3000, Côte-Ste-Catherine, Montréal, Canada H3T 2A7
[b] Centre for Research on e-finance and HEC, 3000, Côte-Ste-Catherine, Montréal, Canada H3T 2A7

Received 3 December 2003; accepted 7 May 2004
Available online 28 July 2004

Abstract

Installment options are Bermudan-style options where the holder periodically decides whether to exercise or not and then to keep the option alive or not (by paying the installment). We develop a dynamic programming procedure to price installment options. We study in particular the geometric Brownian motion case and derive some theoretical properties of the IO contract within this framework. We also characterize the range of installments within which the installment option is not redundant with the European contract. Numerical experiments show the method yields monotonically converging prices, and satisfactory trade-offs between accuracy and computational time. Our approach is finally applied to installment warrants, which are actively traded on the Australian Stock Exchange. Numerical investigation shows the various capital dilution effects resulting from different installment warrant designs.

Keywords: Finance; Dynamic programming; Option pricing; Installment option; Installment warrant

1. Introduction

Installment options (IO) are akin to Bermudan options except that the holder must regularly pay a premium (the "installment") to keep the option alive. The pre-specified dates (thereafter "decision dates") at which the IO may be striked correspond to the installment schedule. Therefore, at each decision date, the holder of the IO must choose between the following

1. to exercise the option, which puts an end to the contract;

* Corresponding author. Tel.: +1 514 340 6490; fax: +1 514 340 5019.
E-mail address: michele.breton@hec.ca (M. Breton).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2004.05.009

2. not to exercise the option and to pay the installment, which keeps the option alive until the next decision date;
3. not to exercise the option and not to pay the installment, which puts an end to the contract.

Among the most actively traded installment options throughout the world presently are the installment warrants on Australian stocks listed on the Australian Stock Exchange (ASX). Installment options are a recent financial innovation that introduces some flexibility in the liquidity management of portfolio strategies. Instead of paying a lump sum for a derivative instrument, the holder of the IO will pay the installments as long as the need for being long in the option is present. In particular, this considerably reduces the cost of entering into a hedging strategy. [1] In addition, the non-payment of an installment suffices to close the position at no transaction cost. This reduces the liquidity risk typically associated with other over-the-counter derivatives.

The aim of this paper is twofold. First, we tackle the problem of pricing IOs using dynamic programming (DP) in a general setting. Second, we investigate the properties of IOs through theoretical and numerical analysis in the Black and Scholes (1973) setting.

Literature on IOs is scarce. Davis et al. (2001, 2002) derive no-arbitrage bounds for the price of the IO and study static versus dynamic hedging strategies within a Black–Scholes framework with stochastic volatility. Their analysis however is restricted to European-style IOs, which allows for an analogy with compound options. Davis et al. (2003) value venture capital using an analogy with IO.

Algorithms based on finite differences have been widely used for pricing options with no known closed-form solution (see e.g. Wilmott et al. (1993) for a survey). Dynamic programming stands as an alternative for low dimensional option pricing. By contrast to finite difference methods, DP does not require time discretization. A DP formulation for pricing American options can be traced back to Chen (1970). He was able to generate theoretical prices directly for a limited number of decision dates. Note however that his paper appeared before the seminal Black and Scholes (1973) contribution and therefore does not apply risk neutral pricing.

Ben-Ameur et al. (2002) show that DP combined with finite elements is particularly well suited for options involving decisions at a limited number of distant dates during the life of the contract. Examples include Bermudan-style options, callables, and convertibles. By construction, IOs allow for both early exercise and installment payment decisions periodically.

The rest of the paper is organized as follows. In Section 2, we develop the model. In Section 3, we solve the Bellman equation and show how the discretization and approximation are made. Section 4 presents the special case of the geometric Brownian motion and derives properties of the value function in this setting. We present numerical illustrations in Section 5. In Section 6 we show how to adapt our approach to the pricing of installment warrants, which are actively traded on the Australian Stock Exchange (ASX). Section 7 concludes.

2. The model

Let the price of the underlying asset $\{S\}$ be a Markov process that verifies the fundamental no-arbitrage property. Let $t_0 = 0$ be the installment option (IO) inception date and $t_1, t_2, \ldots, t_n$ ($t_n = T$) a collection of decision dates scheduled in the contract. An installment design is characterized by the vector of premia $\pi = (\pi_1, \ldots, \pi_{n-1})$ that are to be paid by the holder at dates $t_1, \ldots, t_{n-1}$ to keep the IO alive. The price of the IO is the upfront payment v_0 required at t_0 to enter the contract.

[1] Risk managers may enter the IO contract at a low initial cost and adjust the installment schedule with respect to their cash forecasts and liquidity constraints. This feature is particularly attractive for corporations which massively hedge interest rate and currency risks with forwards, futures or swaps because standard option contracts imply a cost at entry that may be incompatible with a temporary cash shortage.