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Optimization of a large-scale water reservoir network by stochastic dynamic programming with efficient state space discretization

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Abstract

A numerical solution to a 30-dimensional water reservoir network optimization problem, based on stochastic dynamic programming, is presented. In such problems the amount of water to be released from each reservoir is chosen to minimize a nonlinear cost (or maximize benefit) function while satisfying proper constraints. Experimental results show how dimensionality issues, given by the large number of basins and realistic modeling of the stochastic inflows, can be mitigated by employing neural approximators for the value functions, and efficient discretizations of the state space, such as orthogonal arrays, Latin hypercube designs and low-discrepancy sequences.

Keywords: Dynamic programming; Large-scale optimization; Applied probability; Neural networks; Natural resources

1. Introduction

Optimal operation of water reservoir networks has been studied extensively in literature (e.g., [1,12,13,21,34]; Yakowitz [37] provides an excellent survey). Reservoir networks systems can be represented by graphs in which the nodes correspond to water basins and the links are characterized by interbasin transfers. In typical models, the inputs to the nodes are water released from upstream reservoirs and stochastic inflows from external sources (like rivers and rain), while the outputs correspond to the amount of water to be released during a given time period (e.g., a month).

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The dynamics for the single basin can be modeled by a state equation where the amount of water at the beginning of period $t+1$ reflects the flow balance between the water that enters (upstream releases and stochastic inflows) and the water that is released during period t. The amount of water to be released during a given time period from each reservoir is chosen to minimize some possibly nonlinear cost (or maximize benefit) function related to the releases (e.g., power generation), while satisfying proper constraints, e.g., maximum pumpage capacities or target water level in each basin at the beginning of each period t. Thus, optimal management of the reservoir network can be formulated as an optimization problem, in which the aim is to determine the quantity of water releases that minimize a total cost over a given horizon of T time periods (e.g., a year). In the case of large-scale reservoir networks with several basins, nonlinearities and the presence of stochastic variables, the corresponding optimization problem becomes very complex.

This is particularly evident when we want to model the stochastic inflows accurately. In a realistic representation of inflow dynamics, the amount of (random) water flowing into the basins during a given period t depends on the amounts of past periods. An example of realistic modeling, widely and successfully employed in the water resources literature [13,30], is to consider autoregressive linear models of a given order k. To construct this model, the inflows of the past k time periods must be included in the state vector, which consequently can become very large.

Stochastic dynamic programming (SDP) [3,4,28] is the most commonly used solution technique in the reservoir networks management literature [1,12,13,20,27,37]. SDP is based on the definition, at each stage t, of a *value function* which quantifies the cost from that stage through the end of the time horizon. In this way, it is possible to transform the optimization problem into the recursive solution of a sequence of simpler optimization subproblems. It is well known that an exact solution to the SDP equations can be obtained only when system dynamics are linear and the cost (benefit) function is quadratic. In the general case we must seek approximate solutions, which are based on a state space discretization and an approximation of the value functions over the continuous space. Although for the deterministic case there exist efficient versions of dynamic programming, such as Differential DP [19], classical approaches for the stochastic case suffer from the *curse of dimensionality* phenomenon, which is an exponential growth of the computational and memory requirements as the dimension of the state vector increases. This is why only reservoir networks of limited size are considered in the literature, in the absence of restrictive hypotheses on the model. For example, in a recent paper [27], the largest example involved seven state variables. Therefore, for large-scale reservoir networks with accurate inflow dynamics, a very efficient version of SDP is still needed.

In this work we present approximate solutions to a 30-dimensional problem. The state space dimension arises from a test network with 10 reservoirs and inflows modeled by an autoregressive system of order 2, where the cost and the constraints are nonlinear. To the best of the authors' knowledge, this is the largest dimensional problem ever addressed in the reservoirs management literature by SDP techniques, at least without introducing restrictive hypotheses on the cost and/or the model.

For solving such a high-dimensional problem we utilize an approach based on efficient discretization of the state space and approximation of the value functions over the continuous state space by means of a flexible feedforward neural network. Other methods that employ neural networks for reservoir control problems can be found in literature. For instance, in [31], a 10-dimensional model where the cost function represents energy deficit in power generation is solved by an algorithm based on a two-phase neural network.

The approach presented here is based on the high-dimensional continuous-state SDP work of Chen [7–9] and Baglietto et al. [38]. The most commonly employed discretization in the literature consists of a uniform grid of points over the state space, which is not efficient because it is subject to exponential growth. Referring to the area of statistical design of experiments, we consider more efficient discretizations. Specifically, orthogonal arrays (**OA**) [8], OA-based Latin hypercubes