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Financial market dynamics

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Abstract

A necessary precondition for modeling financial markets is a complete understanding of their statistics, including dynamics. Distributions derived from nonextensive Tsallis statistics are closely connected with dynamics described by a nonlinear Fokker–Planck equation. The combination shows promise in describing stochastic processes with power-law distributions and superdiffusive dynamics. We investigate intra-day price changes in the S&P500 stock index within this framework. We find that the power-law tails of the distributions, and the index's anomalously diffusing dynamics, are very accurately described by this approach. Our results show good agreement between market data and Fokker–Planck dynamics. This approach may be applicable in any anomalously diffusing system in which the correlations in time can be accounted for by an Ito–Langevin process with a simple time-dependent diffusion coefficient.

Keywords: Econophysics; Finance; Nonextensive statistics; Stochastic processes

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1. Introduction

In anomalously diffusing systems, a mean-square displacement scales with time according to a power-law, t^α, with $\alpha > 1$ (superdiffusion) or $\alpha < 1$ (subdiffusion). (The case $\alpha = 1$ corresponds to normal diffusion.) Anomalous diffusion has been observed in systems as widely varied as plasma flow [1], surface growth [2], and financial markets [3]. A general framework for treating anomalously diffusing systems is provided by the nonlinear Fokker–Planck equation, which is associated with an underlying Ito–Langevin process [4–6]. This in turn has a very interesting connection to the

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nonextensive entropy proposed by Tsallis [7]: the nonlinear Fokker–Planck equation is solved by time-dependent distributions which maximize the Tsallis entropy [5,8]. This unexpected connection between thermostatistics and anomalous diffusion gives an entirely new way to approach the study of dynamics of anomalously diffusing systems. In this paper, we use this viewpoint to address the dynamics of financial markets.

Several financial markets' indices as well as their member stocks are characterized by price changes whose variances have been shown to undergo anomalous (super) diffusion under time evolution [3,9–11]. Moreover, the probability distributions of the price changes have power-law tails [3,12]. An open long-term question is how best to describe these distributions and their time evolution. Earlier work on currency exchange price changes has shown that the power-law tails can be described by a distribution which maximizes the Tsallis entropy [13]. The connection mentioned above then suggests that the market dynamics might be controlled by a nonlinear Fokker–Planck equation. In this description the power-law tails and the anomalous diffusion arise together quite naturally. Here we show that this approach seems to accurately describe high-frequency intra-day price changes for the S&P500 index. It may also be suitable to describe the superdiffusion and power-law behaviors observed in a broad range of markets and exchanges [3].

What kind of anomalously diffusing system will evolve in time in a nonlinear Fokker–Planck manner? A partial answer is provided by more microscopic underpinnings: a stochastic diffusion equation of Ito–Langevin form. Anomalous diffusion is caused by correlations in time; i.e., a memory effect exists in a market comprised of many interacting investors. Nonlinear Fokker–Planck dynamics result from an Ito–Langevin process in which the diffusion coefficient depends on the probability of the most recent random step x according to $DP(x,t)^{1-q}$. Whenever memory effects can be approximated in this very simple way, then the anomalously diffusing system will obey nonlinear Fokker–Planck dynamics.

A natural question arises: what microscopic interacting models can lead to this behavior? In financial markets, one driving force behind price changes is the mismatch in supply and demand among traders. A number of microscopic Ising-like models have been proposed that attempt to take into account the behavior of interacting traders [14]. But until now there has been no obvious way to decide whether a given model is satisfactory. An essential precondition for developing accurate microscopic interacting many-investor models is a more complete understanding of the markets' statistics, including their dynamics. That is the purpose of this paper.

2. Fokker–Planck dynamics

Because the very many interacting investors in a market exhibit very complex behavior, complete knowledge regarding the interactions and forces at play in the system is unavailable, and perhaps impossible in principle to obtain. A very powerful approach, widely applicable to complex systems with incomplete information, is to maximize an information measure equivalent to the Tsallis form of the entropy [15]. The description we will use was developed in the general context of anomalously diffusing systems