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Efficient heuristic algorithm used for optimal capacitor placement in distribution systems

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ABSTRACT

An efficient heuristic algorithm is presented in this work in order to solve the optimal capacitor placement problem in radial distribution systems. The proposal uses the solution from the mathematical model after relaxing the integrality of the discrete variables as a strategy to identify the most attractive bus to add capacitors to each step of the heuristic algorithm. The relaxed mathematical model is a non-linear programming problem and is solved using a specialized interior point method. The algorithm still incorporates an additional strategy of local search that enables the finding of a group of quality solutions after small alterations in the optimization strategy. Proposed solution methodology has been implemented and tested in known electric systems getting a satisfactory outcome compared with metaheuristic methods.

The tests carried out in electric systems known in specialized literature reveal the satisfactory outcome of the proposed algorithm compared with metaheuristic methods.

1. Introduction

In the radial distribution systems, shunt capacitors must be installed in order to compensate the reactive power of the loads and reduce the reactive power provided by the system, to diminish energy losses, to improve the voltage magnitude profile and, on a smaller scale, to release system capacity [1–5]. Thus, the general problem of optimal capacitor placement (OCP) appears in radial distribution systems. This problem consists in determining the *optimal number*, *locations*, *types*, *sizes* and *switching times*, for the case of switched capacitors, in order to maximize costs savings while maintaining requested good operation conditions.

This problem is interesting because its mathematical model is a mixed integer non-linear programming problem (MINLP) with a non-differentiable objective function due to the fact that the costs of the capacitor vary in a discrete manner. The system load also varies continuously throughout the day. In this way, practically all the research related to the OCP assumes the demand variation in a determined number of discrete load levels in order to solve the problem using existing optimization techniques. Usually the demands have three load levels, that is, peak load, medium load, and light load. Therefore, for each load level, the control scheme of the switched capacitors must be determined.

The OCP has been carefully analyzed in the mathematical model as well as in the development of solution techniques. At the early stage, simplified network models that allowed the use of optimization algorithms were used. The mathematical modeling was improved and a greater contribution in this aspect was carried out in Ref. [1] where an optimization mathematical model that is nowadays used by the researchers is shown.

The optimization techniques proposed to solve the OCP problem, nowadays, can be grouped into three large groups: (1) analytical methods, (2) numerical programming algorithms, (3) approximate algorithms. Among numerical programming algorithms, methods such as Benders decomposition techniques and the Branch and Bound algorithms, used in the initial phases but limited to small systems and simplified mathematical models [1,6], can be included. The approximated algorithms can be grouped into two groups: (i) heuristic algorithms and (ii) metaheuristics.

Heuristic algorithms generally use a performance criterion or a sensitivity indicator incorporated into an optimization strategy in order to find quality solutions for complex problems [7–9]. These algorithms present the advantage of finding quality solutions with small processing efforts and are generally robust and simple to understand and to implement. These algorithms differ in two

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Nomenclature

nr	number of electric system branches
nb	number of system buses
nc	number of candidate buses to place capacitors
k	bus index, or line index between bus k and $k+1$
i	operation level index, $i = 0, \ldots, nt$
x^i	vector of decision variables
$\hat{u}^i$	vector of control variables, capacitors size as discrete variable
$B^i(x^i, \hat{u}^i) = 0$	system power flow equations at load level i
$H^i(x^i, \hat{u}^i) \leqslant 0$	system operation constraints at load level i
$\hat{u}^0$	vector of rated capacitor kvars installed in the system
$u^{\max}$	vector of superior limit of $\hat{u}^0$
$\hat{u}^0_k$	rated capacitor kvar placed at bus k
$\hat{u}^i_k$, $\hat{u}^i_f$	switched and fixed capacitor operation point placed at bus k, respectively
k_e, rc	energy cost in $/kWh and capacitor purchase cost $/kVAr, respectively
c	capacitor installation cost $
S^i	load duration curve factor at load level i
V^i_k	voltage magnitude at bus k for load level i
P^0_{Lk}, Q^0_{Lk}	real and reactive nominal load at bus k, respectively
P^i_k, Q^i_k	real and reactive power flow leaving bus k and entering bus $k+1$, at load level i, respectively
Qe^i_{k+1}	reactive injection from existing capacitors at bus $k+1$ and for load level i
V_{min}, V_{max}	minimum and maximum values allowed for V^i_k, for all bus k, and for all load level i, respectively

fundamental aspects: (1) the performance criterion used and (2) the proposed optimization strategy. The performance criterion can be a very simple decision or it can be obtained by solving a complex subproblem that, in many cases, implies in solving the real problem itself after relaxing the characteristics of some variables and/or constraints.

Metaheuristics have been the most used algorithms in the last few years to solve complex problems in the field of operational research. Recent research using metaheuristics shows the great popularity of these techniques to solve OCP problems [2,5,10–13].

The mathematical model proposed in [1] was used in this work, which is the most used model for the OCP, and a heuristic algorithm was presented to solve this problem. Our proposal uses an interior point method to solve a non-linear programming problem (NLP), yielded after relaxing integer variables of the original MINLP problem, as an indicator to add or place new capacitors in the most attractive bus of the system. This process is controlled by an constructive heuristic algorithm.

Another proposal presented in this work is to leave the substation voltage (S.S. Volt.) as a decision variable, finding better quality solutions when compared to the optimization solutions that set the magnitude voltage of the substation to the nominal value.

Our experience shows that in highly stressed systems, that is, with elevated violation in the voltage magnitude, it is very difficult to find a feasible solution only adding capacitors. Additionally, in case these feasible solutions exist, they generally represent operation points where losses are greater than in the case of operation without capacitors. In this context, the capacitors diminish the real energy losses and improve the voltage magnitude profile but do not have the capacity to correct the voltage magnitude in highly charged systems. It must be observed that almost all the metaheuristics presented in specialized literature keep the value of the substation voltage fixed [1,4,14,15].

2. Mathematical formulation

In the mathematical modeling, the decision variables were separated into two groups, x^i and $\hat{u}^i$. Thus, the group of decision variables are gathered in vector $y^i = [x^i; \hat{u}^i]^t$ for any load level i. The mathematical model used in this work deals with a horizon of operation of T years, and is as follows [1]:

$$\min \ \ v = k_e \sum_{i=0}^{nt} T^i p^i(y^i) + \sum_{k=1}^{nc} f(\hat{u}^o_k) \tag{1}$$

$s.t.$

$$B^i(x^i, \hat{u}^i) = 0; \quad i = 0, 1, \ldots, nt$$
$$H^i(x^{i,}, \hat{u}^i) \leqslant 0; \quad i = 0, 1, \ldots, nt$$
$$0 \leqslant \hat{u}^0 \leqslant u^{\max};$$
$$0 \leqslant \hat{u}^i_{Ck} \leqslant \hat{u}^o_k;$$
$$0 \leqslant \hat{u}^i_{fk} = \hat{u}^o_k;$$
$$\hat{u}^i = \left[\hat{u}^i_c \hat{u}^i_f\right] \text{ discrete.}$$

In (1) the objective function to be minimized is v and presents two conflicting parts: the operation losses cost $p^i(y^i)$ at time T^i and the acquisition and installation capacitor costs $f(\hat{u}^0_k)$, this last as a discrete function.

The main differences from other researches presented in the specialized literature are as follows: bus voltage must remain within the limits specified by the companies that regulate electric energy, fixed and/or switched capacitors can be placed at different costs and according actual network scenario, reasonably elevated load factor during the peak hour can be considered. This last, causes high magnitude voltage drops producing an unfeasible operation of the system without the installation of the capacitors and appropriate coordination of voltage substation regulator.

It is assumed that the load duration curve comes in $(nt+1)$ discrete operation levels and the uniform load variation in the system buses was also considered. However, the modeling and the proposed solution can work with no problems with other demand variations in any system bus, as well as for a greater number of discrete demand levels. In this case, the fact that system loads are balanced was also considered.

2.1. Relaxed mathematical modeling

The proposed algorithm solves, in each step, a NLP that is a relaxed version of the problem (1) and consists in treating the discrete variable $\hat{u}$ as a continuous one u. The solution for this NLP allows the identification of the most attractive bus in which discrete size capacitors must be added in the process controlled by the constructive heuristic algorithm (CHA). In order to simplify the mathematical model presentation, the following relation for line k, between bus k and $k+1$ has been defined:

$$\beta^i_k = \left({P^i_k}^2 + {Q^i_k}^2\right) / {V^i_k}^2. \tag{2}$$

Then, in (1) the resistive system losses can be found by using the following relation [1,5]: