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Measuring systemic risk using vine-copula

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ABSTRACT

We present an intuitive model of systemic risk to analyse the complex interdependencies between different borrowers. We characterise systemic risk by the way that financial institutions are interconnected. Using their probability of default, we classify different international financial institutions into five rating groups. Then we use the state-of-the-art canonical (C-) and D-vine copulae to investigate the partial correlation structure between the rating groups. Amongst many interesting findings, we discover that the second tier financial institutions pay a larger contribution to the systemic risk than the top tier borrowers. Further, we discuss an application of our methodology for pricing credit derivative swaps.

1. Introduction

A major shortcoming in many of the models underlying the financial system is their inadequacy to fully comprehend the risk in extreme events. Not only is it essential that we concern ourselves with these unlikely events in isolation, or tail risk, but it is becoming increasingly evident that we should also concern ourselves with these unlikely events in tandem, or systemic risk. As the recent financial crisis illustrates, tail and systemic risk are very real and very devastating.

The copula is a mathematical tool for modelling the joint distribution of simultaneous events. From the perspective of tail and systemic risk, the copula is interesting in that it allows us to decouple the marginal distribution (that which is associated with tail risk) from the dependence structure (that which is associated with systemic risk) and model each separately with a greater degree of precision. Nevertheless, the problem in practical applications is how to identify this copula. For the bivariate case, a rich variety of copula families are available and well-investigated (see Brechmann et al., 2013; Brechmann and Joe, 2014). The use of copula is however challenging in higher dimensions, where standard multivariate copulae suffer from rather inflexible structures. Vine copulae overcome such limitations and are able to model complex dependency patterns by benefiting from the large range of bivariate copulae as building blocks.

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1.1. Background

Since the global financial crisis (GFC) in 2008, there has been a strong interest in the development of accurate models for the dependency structure of default risk between financial institutions. The aftermath of the failure of Bear Stearns and Lehman Brothers, presented a significant *systemic risk* due to the interdependency of financial institutions across the globe. The impact of the global financial crisis reached its peak in Europe in July 2015, when Greece was placed in arrears on its public debt to the International Monetary Fund. This caused indexes worldwide to tumble, as many are now uncertain about Greece's future, fearing a potential exit from the European Union. Although current financial regulations have attempted to manage the systemic risk through the Basel capital requirements, their actions are micro-prudential in nature, in that they seek to limit each institutions risk. However, unless the external costs of systemic risk are internalized by each financial institution, the institution will have the incentive to take risks that are borne by others in the economy. Therefore, it is necessary for banks and other financial institutions to develop an active and market based management of credit risk.

It is known that the individual defaults do not have a significant impact on the risk of a bank's portfolio since they are (typically) well diversified. However, a portfolio of a large number of small loans, with systemic dependencies, is perceived to be very risky. To manage this risk, the conventional approach is to hedge the counter party risk, using basket credit derivatives and collateralised debt obligations. As more financial firms try to manage their credit risk at the portfolio level, the demand for basket credit derivative products will most likely continue to grow.

Therefore, the problem of default correlation is central to the valuation of credit derivatives, even in the case of a simple credit default swap with one underlying reference asset. After the 1997–98 financial crisis in Southeast Asia,[1] which provided sufficient evidence of the default correlation between financial institutions and credit derivatives, a body of literature emerged recommending the use of Gaussian copula to model joint distribution of the probability of default. Li (1999) pioneered the use of Gaussian copula models for the pricing of collateralised debt obligations. The simplicity and the elegance of the approach soon drew attention in academia and amongst market practitioners in the valuation of credit derivatives. For example, in a comprehensive study, Das and Geng (2004) analysed the joint default process of hundreds of issuers, using different copula functions including normal, Gumbel, Clayton and Student *t* copulae. In a more recent paper, J. Chen et al. (2014) employed a copula model with stochastic correlation for pricing credit derivatives portfolios, under the assumption that the systematic factor and idiosyncratic factors subject to the fat-tailed, mixed G-VG distribution instead of the traditional Gaussian distribution.

Copula models have also been used for modelling the dependency structure of other financial assets. For instance, Bhatti and Nguyen (2012) used the conditional extreme value theory and time-varying copulae to capture the tail dependence between selected international stock markets. Naifar (2012) modelled the dependence structure between risk premium, equity return and volatility in the presence of jump-risk. Fenech et al. (2015) discussed loan default correlation using an Archimedean copula approach. Nguyen and Bhatti (2012) used nonparametric chiKendall plots and semi parametric copula to capture the dependency between oil prices and stock markets.

The popularity of copula models is due to the implication of Sklar's theorem (see Sklar, 1959) that the modelling of the marginal distributions can be separated from the dependence modelling in terms of the copula. More importantly, copulae provide a convenient framework for measuring the extreme dependence between two random variables. In a crisis, financial correlations typically increase (see studies by Das et al. (2007) and Duffie et al. (2009)); hence, it would be desirable to apply a correlation model with high co-movements in the lower tail of the joint distribution. However, the identification of the copula families for problems with higher dimensions than the bivariate cases remains as a major problem. Brechmann and Schepsmeier (2013) demonstrates that standard multivariate copulae such as the multivariate Gaussian or Student-*t*, as well as exchangeable Archimedean copulae lack the flexibility of accurately modelling the dependence amongst larger numbers of variables. Generalisations of these offer some improvement, but typically become rather intricate in their structure and hence exhibit other limitations such as parameter restrictions.

1.2. Vine copulae

To avoid all of these problems, in this paper we propose the use of *vine copulae* for more accurate modelling of the dependence amongst a larger number of variables. Vine copulae were initially proposed by Joe (1996) and further developed in Bedford and Cooke (2001), Bedford and Cooke (2002) as well as in Kurowicka and Cooke (2006). They use a cascade of bivariate copulae, known as *pair-copulae*, to build multivariate copulae, such that a multivariate probability density can be decomposed into bivariate copulae. Since each pair-copula is chosen independently, it provides a significantly more flexible framework for dependence modelling in credit risk. Most importantly, correlation asymmetries and *tail dependencies* can be taken into account to build more parsimonious models. In summary, vines combine the flexibility of bivariate copulae and the advantages of multivariate copula modelling, that is separation of marginal and dependence modelling.

[1] Mansor et al. (2015) provides a comprehensive discussion on the 1997–98 Asian financial crisis and 2008–09 GFC, in the context of mutual fund performance.

Aas et al. (2009) introduced vine copulae into the finance and insurance literature, where they also described statistical inference techniques for the two classes of canonical C- (and D-) vines. Despite the material improvement that vine copulae provide, they have been rather overlooked in the credit risk literature. In this paper, for the first time we study the *partial correlation* structure by using vine copula. We propose that the partial correlation vine fully characterises the correlation structure of joint distribution, without having to assume *independence* between the variables (i.e., they may be related). Unlike the values in a correlation matrix, the partial correlation in a vine need not satisfy an algebraic constraint like positive definiteness. Kim et al. (2011) developed a robust statistical framework for the analysis of partial correlation with Gaussian vine copulae. In this paper, we take a similar approach, however, we also consider Gumbel, Clayton and student *t* copula in joint dependence relationship, so that we can capture the stylised features of the financial data.

The remainder of this paper is structured as follows. In Section 2 we thoroughly describe the data on probability of default (PD) for financial institutions. Section 3 provides a discussion on empirical features of dependence in the joint distribution, which serves as the motivation for our choice of models. Section 4 provides the theoretical framework for vine copulae and Section 5 presents the empirical analysis and a thorough discussion on the model applications. Section 6 concludes the paper.

2. Data description

Our data set comprises global financial institutions, including banks and insurance companies, tracked by Thomson Reuters every month during the period of Jan 2005 to Jan 2015. For each issuer, we have obtained PDs based on Thomson Reuters' structural model.

Thomson Reuters evaluates the equity market's view of credit risk via a propitiatory structural default prediction framework based on the Merton model (Merton, 1974) which models a company's equity as a call option on its assets. In this framework, the probability of default (PD) equates to the probability that the option expires worthless. Thomson Reuters produces daily updated estimates of the probability of default or bankruptcy within one year for 35,000 companies globally, including financials. The default probabilities are also mapped to letter ratings and ranked to create 1–100 percentile scores.

We consider 60 traded global financial institutions (banks and insurance companies), where we classify issuers into five credit rating groups. The PDs within these classes vary from high to low. Table 1 shows the sorting classification and Table 2 reports the descriptive statistics of our data from rating classes 1 through 5. In Table 2 we can observe that the mean and the standard deviation increase from the first rating class to the fifth rating class, where changes tend to be higher for lower-grade issuers. Table 3 presents the dependence between rating classes measured by Kendall's τ statistic, a means of determining the dependence between any two time series. Fig. 1 pictorially illustrates the linear correlation of each rating classes, where each edge is based on weights of Kendall's τ amongst two rating classes. Specifically, a higher correlation is presented by darker and thicker lines, and a lower correlation is presented by lighter and thinner lines (Fig. 1).

Table 1
Rating class the database comprises five rating categories.

Rating class	Rating	Credit group
1	Aaa, Aa	High grade
2	A	High grade
3	Baa	Medium grade
4	Ba, B	Medium grade
5	Caa, Ca, C	Low grade