Journal of Banking & Finance 37 (2013) 3169–3180

Contents lists available at SciVerse ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Systemic risk measurement: Multivariate GARCH estimation of CoVaR ☆

Giulio Girardi [a,*], A. Tolga Ergün [b]

[a] Division of Risk Strategy and Financial Innovation, US Securities and Exchange Commission, Washington, DC, United States
[b] State Street Corp., Boston, MA, United States

ARTICLE INFO

Article history:
Received 5 July 2011
Accepted 26 February 2013
Available online 7 March 2013

JEL classification:
G11
G21
G32
G38

Keywords:
Value-at-Risk
Conditional Value-at-Risk
Systemic Risk
DCC model

ABSTRACT

We modify Adrian and Brunnermeier's (2011) CoVaR, the VaR of the financial system conditional on an institution being in financial distress. We change the definition of financial distress from an institution being exactly at its VaR to being at most at its VaR. This change allows us to consider more severe distress events, to backtest CoVaR, and to improve its consistency (monotonicity) with respect to the dependence parameter. We define the systemic risk contribution of an institution as the change from its CoVaR in its benchmark state (defined as a one-standard deviation event) to its CoVaR under financial distress. We estimate the systemic risk contributions of four financial industry groups consisting of a large number of institutions for the sample period June 2000 to February 2008 and the 12 months prior to the beginning of the crisis. We also investigate the link between institutions' contributions to systemic risk and their characteristics.

Published by Elsevier B.V.

1. Introduction

The recent financial crisis has alerted the public to the fragility of the financial system and systemic risk. Value-at-Risk (VaR), arguably the most widely-used risk measure by financial institutions, has been criticized by many as incapable of capturing the systemic nature of risk since its focus is on an institution in isolation. VaR has been used by regulators as an instrument to determine capital levels that need to be set aside by financial institutions against market risks. However, since VaR considers only the risk that an institution faces when considered in isolation, it is not possible to gauge the risk facing the financial system from an institution's VaR. As a result, recently, there has been considerable interest in alternative risk measures which do not suffer from VaR's shortcoming, namely, its inability to account for the possibly systemic nature of an institution's risk and financial distress.

One of these recent studies is Adrian and Brunnermeier (2011) (referred to as AB henceforth) who introduce a new risk measure: Conditional Value-at-Risk (CoVaR). They define $\text{CoVaR}^{i|j}$ as the VaR of institution i conditional on institution j being in financial distress, which they define as institution j being at its VaR. By conditioning on another institution's financial distress, they aim to go beyond idiosyncratic risk and to capture possible risk spillovers among financial institutions.

While the $\text{CoVaR}^{i|j}$ measure can be computed for any two financial institutions i and j, AB consider the specific case where i is the financial system. In this case CoVaR becomes the VaR of the financial system conditional on institution j being in financial distress, and hence can be used to determine a financial institution's contribution to systemic risk. While it may not be easy to find consensus on the exact definition of systemic risk, the following quote from the Federal Reserve Governor Daniel Tarullo's July 2009 testimony before the Senate Banking, Housing, and Urban Affairs Committee is one definition that many can probably agree on, and also one that CoVaR seems to capture[1]

> "*Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy.*"

Among other recent studies that propose measures to quantify systemic risk are Billio et al. (2012), Zhou (2010), Huang et al. (2009), Segoviano and Goodhart (2009), Acharya et al. (2010),

☆ The Securities and Exchange Commission, as a matter of policy, disclaims responsibility for any private publication or statement by any of its employees. The views expressed herein are those of the author and do not necessarily reflect the views of the Commission or of the author's colleagues on the staff of the Commission.

* Corresponding author. Tel.: +1 857 277 3475.
E-mail addresses: girardig@sec.gov (G. Girardi), aergun2@statestreet.com (A. Tolga Ergün).

[1] http://www.federalreserve.gov/newsevents/testimony/tarullo20090723a.htm.

0378-4266/$ - see front matter Published by Elsevier B.V.
http://dx.doi.org/10.1016/j.jbankfin.2013.02.027

Brownlees and Engle (2011), and Allen et al. (2010). Billio et al. (2012) use principal components analysis and Granger-causality tests to propose several econometric measures of systemic risk to capture interconnectedness among the returns of hedge funds, banks, brokers, and insurance companies. Zhou (2010) uses multivariate Extreme Value Theory framework to provide two measures of systemic risk: the systemic impact index and the vulnerability index. The former assesses the risk that an institution imposes on the system and the latter the risk that the system imposes on the institution. Huang et al. (2009) use data on credit default swaps (CDSs) of financial institutions and equity return correlations to model systemic risk as the price of insurance against financial distress. Segoviano and Goodhart (2009) work with CDS data too, and measure institutions' contributions to the distress of the financial system within a multivariate setting. Acharya et al. (2010) use equity returns of financial institutions to calculate Systemic Expected Shortfall (SES) and Marginal Expected Shortfall (MES). MES is an institution's average loss when the financial system is in its left tail, and SES is calculated as the weighted average of the institution's MES and its leverage. While Acharya et al. (2010) calculate time-invariant MES measures, Brownlees and Engle (2011) compute their time-series by using a bivariate GARCH model and non-parametric tail estimators. Finally, Allen et al. (2010) propose a measure of aggregate systemic risk (CATFIN) which, as opposed to the micro-level systemic risk measures such as CoVaR and MES, gauges the macroeconomic effects of systemic risk taking in the banking system as a whole.

When we compare CoVaR with the MES measure of Acharya et al. (2010) and Brownlees and Engle (2011), there is a difference in terms of the conditioning event and the direction; while MES looks at the returns of an institution when the financial system is in distress and experiencing losses, CoVaR does the opposite and looks at the returns of the financial system when an institution is in financial distress. This difference arises not because of the intrinsic properties of the two measures, but because of the usage that has been done of the two. In fact, for both measures it is possible and straightforward to reverse the analysis. In that case CoVaR would correspond to the VaR of an institution conditional on the financial system being in distress, i.e., being at its VaR. This reverse CoVaR would be more in the spirit of MES as it would be measuring the exposure of an institution to the distress of the financial system.[2] However, in the systemic risk definition given in the quote above, it is the failure of an institution that is the cause of distress for the financial system. Therefore, like AB, we only consider CoVaR$^{i|j}$ where i is the financial system, and condition on the financial distress of an institution j.

In this study, we generalize the definition of CoVaR proposed by AB by assuming that the conditioning financial distress event refers to the return of institution j being *at most* at its VaR ($R^j \leqslant \text{VaR}^j$) as opposed to being *exactly* at its VaR ($R^j = \text{VaR}^j$). This change allows us to consider more severe distress events of institution j that are farther in the tail (below its VaR) and to backtest CoVaR estimates using the standard Kupiec (1995) and Christoffersen (1998) tests used for backtesting VaR. In addition, this change also improves the consistency of CoVaR with respect to the dependence parameter; Mainik and Schaanning (2012) show that when financial distress is defined as proposed in our work ($R^j \leqslant \text{VaR}^j$), for a wide range of models, CoVaR has a monotonic relation with the dependence parameter.[3] In other words, as the institution becomes more and more correlated with the financial system, its systemic risk increases. On the other hand, when financial distress is defined as in AB ($R^j = \text{VaR}^j$), CoVaR counter-intuitively starts decreasing after the dependence parameter crosses a threshold. As Mainik and Schaanning argue, in this case CoVaR fails to detect systemic risk where it is most pronounced (high degree of dependence) and financial regulation based on CoVaR with this distress definition could introduce additional instability and set wrong incentives. Lastly, due to the time-varying correlation of the GARCH model, the CoVaR of an institution here has a time-varying exposure to its VaR which, by construction, is not the case in Adrian and Brunnermeier (2011). This feature allows the possible changes over time in the linkage between the institution and the financial system to be captured and incorporated in the systemic risk measure.

We define the systemic risk contribution of an institution as the change from its CoVaR when the institution is in its benchmark state to its CoVaR under financial distress. We define the benchmark state as a one-standard deviation about the mean event. Using daily data from June 2000 to February 2008 for a large number of financial institutions from four industry groups (depositories; insurers; broker-dealers; and others, which are non-depository institutions including government sponsored enterprises), we obtain the industry groups' time-varying CoVaR estimates. We use a three-step procedure to obtain the CoVaR estimates: in the first step we estimate the VaR of each financial institution; in the second step we use the bivariate DCC model of Engle (2002) to estimate the joint distribution of the financial system-institution pair for each institution; and in the third step we numerically solve for CoVaR. To take both skewness and kurtosis into account, we estimate the GARCH models (univariate in the first step and bivariate in the second step) using skewed-*t* as well as the Gaussian distribution. We backtest the CoVaR estimates retrieved from our three-step procedure. We also investigate the link between institutions' characteristics such as size, leverage, and equity beta and their contributions to systemic risk by running panel regressions. Finally, using 12 months of data prior to the beginning of June 2007, we compute each industry group's pre-crisis systemic risk contribution.

Backtesting results show the importance of taking skewness and kurtosis into account as the CoVaR estimates based on the Gaussian distribution fail to satisfy the unconditional coverage property and hence are rejected. CoVaR estimates based on the skewed-*t* distribution, on the other hand, satisfy both the unconditional and conditional coverage properties. During the sample period June 2000 to February 2008, depository institutions were the largest contributors to systemic risk followed by broker-dealers, insurance companies, and non-depository institutions. Unlike in AB, our systemic risk measure and VaR have a weak relation not only in the cross-section but also in the time-series. Thus, the time-series of the systemic risk measure here potentially could have information that is different from the information in the time-series of the institution's VaR. From a regulatory perspective this suggests that monitoring a firm's tail risk may not be sufficient to forecast its systemic risk contribution. Institutions' leverage, size, and beta play an important role in explaining their contributions to systemic risk. Our pre-crisis analysis shows that the systemic risk of all industry groups increased substantially during the 12 month period prior to the beginning of June 2007. Prior to the crisis, while depository institutions were systemically the most risky followed by broker-dealers as in our large sample, the gap was almost closed.

The remainder of the paper is organized as follows: Section 2 formally defines CoVaR and presents the three-step procedure to estimate it. Section 3 describes the maximum likelihood estimation and the backtesting procedure. Section 4 presents the data and estimation results for the June 2000 to February 2008 sample. Section 5 has the results of the subsample analysis using

[2] As AB mention, CoVaR is directional; there is no reason to expect CoVaR of the system conditional on an institution to be the same as CoVaR of the institution conditional on the system. See AB for other properties of CoVaR.

[3] The models they study are when the financial system - institution pair has an elliptical bivariate distribution such as bivariate Gaussian or bivariate-*t*, has bivariate distributions with Gaussian and *t* copulas, and has a bivariate Gumbel copula with *t* marginals.