Computers & Industrial Engineering 59 (2010) 509–519

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Heuristic algorithms for preemptive scheduling in a two-stage hybrid flowshop with additional renewable resources at each stage ☆

Ewa Figielska *

Warsaw School of Computer Science, Lewartowskiego 17, 00-169 Warsaw, Poland

ARTICLE INFO

Article history:
Received 23 January 2008
Received in revised form 4 June 2010
Accepted 10 June 2010
Available online 15 June 2010

Keywords:
Scheduling
Flowshop
Parallel machines
Resource constraints
Heuristics
Linear programming

ABSTRACT

This paper deals with the problem of preemptive scheduling in a two-stage flowshop with parallel unrelated machines and renewable resources at both the stages. The resource requirements are of a 0–1 type. The objective is the minimization of makespan. The problem is NP-hard. Four heuristic algorithms using linear programming are proposed for solving this problem. Performance of the algorithms is analyzed experimentally by comparing heuristic solutions with the lower bound on the optimal makespan. Statistical comparative analysis of the developed algorithms is carried out. The results of a computational experiment show that the proposed algorithms are able to produce good quality solutions in a small amount of computation time.

1. Introduction

This paper deals with the problem of preemptive scheduling in a two-stage flowshop with parallel unrelated machines and additional renewable resource constraints at each stage. The resource requirements are of a 0–1 type. The objective is to minimize the makespan.

The problem of resource constrained scheduling of parallel machines has been widely investigated in the literature (de Werra, 1984, 1988; Figielska, 1999; Słowiński, 1980, 1981). However resource constraints have not been considered until recently in the context of scheduling in the multiprocessor flowshop environment. The multiprocessor flowshop is a system which consists of a set of two or more processing centers (processing stages) with at least one center having two or more parallel machines. A job in such a system is processed successively at processing stages, and all jobs pass through the stages in the same order. At a stage with parallel machines a job can be processed on any machine and a machine can work on at most one job at a time. The multiprocessor flowshop scheduling problems have received considerable attention from researchers in recent times. Most literature in this area addresses problems with identical parallel machines and nonpreemptive jobs, and include among others Gupta (1988), Chen (1995), Haouari and M'Hallah (1997), Brah and Loo (1999), and Linn and Zhang (1999). In our paper we focus on preemptive scheduling in a multiprocessor flowshop with unrelated machines (known as a hybrid flowshop) and additional renewable resources, where jobs require, besides machines, additional resources which are available in limited quantities at every moment.

As mentioned earlier, the flowshop scheduling problem with additional renewable resources has not been studied in the literature until recently. In Figielska (2008) additional renewable resources have been taken into consideration only at the first stage of the hybrid flowshop with a single machine at the second stage. In this paper, we extend the method proposed in Figielska (2008) to the case of the flowshop with parallel unrelated machines and resource constraints at two stages. We propose four heuristic algorithms based on linear programming for solving the scheduling problem in this case. While solving the problem at the first stage, these algorithms, for the selection of jobs to be processed in parallel during successive periods of time, use selection rules appropriate for finding a minimal makespan in the whole flowshop. At the second stage, similarly as at the first stage, there are a number of parallel unrelated machines and the jobs to be processed require, besides machines, additional resources which are available in limited amounts at every moment. Each job is ready to be processed at the second stage at the moment of completing its processing at the first stage. So, given the schedule at the first stage, we have to solve the minimum makespan resource constrained scheduling problem with parallel unrelated machines and job dependent ready times (the ready times are equal to completion times at stage 1). For solving this problem to optimality, we develop a procedure based on

☆ This manuscript was processed by Area Editor John W. Fowler.

* Tel.: +48 22 4361949.
E-mail address: efigielska@poczta.wwsi.edu.pl

0360-8352/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cie.2010.06.006

linear programming. This procedure first determines processing times during which jobs (or parts of jobs) are executed on machines in all the time intervals between successive ready times, and then it constructs in these intervals subschedules satisfying resource constraints at every moment. The schedule in the flowshop consists of the schedule at the first stage and the schedule at the second stage. The makespan of the schedule is equal to the maximum job completion time at the second stage.

Performance of the algorithms is analyzed experimentally by comparing the heuristic solutions with the lower bound on the optimal makespan. Statistical comparative analysis of the developed algorithms is carried out.

The problem of scheduling in a multiprocessor flowshop arises in real-life systems that are encountered in a variety of industries, e.g. in chemical, polymer, petrochemical industries (Salvador, 1973). The multiprocessor scheduling problem is also met in computer systems and telecommunication networks (Brah, 1988). In the multiprocessor flowshop there are stages with parallel machines. At each stage with parallel machines jobs can be processed through any one of these machines. Because jobs that are simultaneously processed on parallel machines may use the same resource, the problem of resource constrained scheduling arises when the amount of the available resource is limited. This takes place when, for example, the number of workers attending the machines, or the number of tools that are used by simultaneously executed jobs, is limited. Resource requirements of a 0–1 type can be met, for example in computer systems in which one peripheral device (e.g. a printer) is additionally required to perform a job (Kellerer & Strusevich, 2002). The problems with preemptive jobs are common in the mass production of a large number of products which can be processed in parts or when an article is produced in a great amount, for example in the textile industry (Serafini, 1996) where the processing of any job (the article to be woven) on one of the parallel machines (the looms) may be interrupted (preempted) and resumed on the same or the other machine.

The remainder of this paper is organized as follows. The next section contains the description of the problem we deal with. In Section 3, the heuristic algorithms are presented. An illustrative example is given in Section 4. In Section 5, a lower bound is derived. Results of a computational experiment are reported and discussed in Section 6. Section 7 summarizes the paper.

2. Problem description

The problem being considered can be formally described as follows. There are n preemptive jobs to be processed at two stages in the same technological order, first at stage 1 then at stage 2. There are m_1 and m_2 parallel unrelated machines, respectively, at stage 1 and at stage 2. A job upon finishing its processing at stage 1 is ready to be processed at stage 2; it may be processed at stage 2 when a machine is available there, or it may reside in a buffer space of unlimited capacity following stage 1 until one of the machines at stage 2 becomes available. At each stage, a job can be processed on any of the parallel machines, and its processing times may be different on different machines. The processing times of job j are equal to p_{ij1} and p_{ij2} if it is executed on machine i at stage 1 and stage 2, respectively. The same job cannot be processed on the multiple machines at the same time, and no machine can work on more than one job at the same time. The processing of a job on a machine can be interrupted at any moment and resumed later on the same or another machine of the same stage. It is assumed that there is no time penalty for preempting a job. Jobs for their processing at both stages, besides machines, require additional renewable resources. All required resources are granted to a job before its processing begins or resumes and they are returned by the job after finishing its processing at a stage or in the case of its preemption. We assume that resources are not shared by the stages, and that at stages 1 and 2 there are available, respectively, the sets R_1 and R_2 of resource types. A resource of type r is available in amounts limited to W_r^1 and W_r^2 units at a time, if it belongs to R_1 and R_2, respectively. The total usage of a resource at any moment by jobs simultaneously executed on parallel machines at a stage cannot exceed the availability of this resource at this stage. Each job, during its processing at a stage, uses 0 or 1 unit of the resource of each type specific to this stage. The objective is to find a schedule which minimizes makespan in the flowshop, C_{max}, which is equal to the maximum job completion time at stage 2.

This problem is NP-hard in the strong sense since the simpler problem of preemptive scheduling in the two-stage flowshop without resource constraints, with two identical parallel machines at one stage and one machine at another is NP-hard in the strong sense (Hoogeveen, Lenstra, & Veltman, 1996).

Using standard three-field notation for resource constrained scheduling problems given in Błażewicz, Lenstra, and Rinnoy Kan (1983) and Graham, Lawler, Lenstra, and Rinnooy Kan (1979) and extended to hybrid flow shop scheduling problems in Vignier, Billaut, and Proust (1999), we denote the problem under consideration by FH2|$pmtn, res \cdots 1$|C_{max}. Here "FH2" stays for a two-stage hybrid flowshop, "*pmtn*" says that preemptions are allowed, "*res* $\cdots$ 1" implies that the number of resources and the total amount of each resource available at any time are parts of the input and each job uses no more than 1 unit of a resource at a moment, "C_{max}" stays for the maximization of the makespan.

3. Heuristic algorithms

The heuristic proposed in this paper for the minimum makespan problem of preemptive scheduling in a two-stage flowshop with parallel unrelated machines and resource constraints at both stages, first solves the resource constrained unrelated machines scheduling problem at the first stage, and then, using the solution of this problem, it solves to optimality the resource constrained unrelated machine scheduling problem at the second stage. The makespan in the flowshop is equal to the sum of the makespan at stage 2 and the minimum completion time at stage 1.

3.1. Finding a schedule at stage 1

To solve the problem at the first stage, we use the algorithm proposed in Figielska (2008) (given in detail in Appendix B) to which we introduce the selection rules (see Eqs. ((1)–(4))) appropriate for the flowshop with unrelated machines at both stages. The algorithm first finds the optimal length of the schedule, T^*, and the optimal processing times, x_{ij}^*, during which jobs (or parts of jobs) are executed on machines, by solving an LP problem ((A1)–(A7) in Appendix A) where resource constraints at any moment are replaced by resource constraints over the whole time T^*. Then, a schedule is constructed where resource constraints are satisfied at every moment (see Appendix B).

Before the construction of the schedule, weights, w_j ($j = 1, \ldots, n$), of all the jobs are determined. In this paper, we determine the weights of jobs according to the following four rules.

The 1st rule:

$$w_j = \text{a random number from } U[0, 1) \tag{1}$$

The 2nd rule:

$$w_j = \frac{\min_{k=1,\ldots,n}\{Z_k\}}{Z_j} \tag{2}$$

The 3rd rule: