Computers & Industrial Engineering 62 (2012) 276–285

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

# Exact and heuristic algorithms for the aerial refueling parallel machine scheduling problem with due date-to-deadline window and ready times ☆

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ARTICLE INFO

*Article history:*
Received 12 January 2011
Received in revised form 5 August 2011
Accepted 23 September 2011
Available online 29 September 2011

*Keywords:*
Parallel machine scheduling
Total weighted tardiness
Due date-to-deadline window
Ready time
Dispatching rule
Simulated annealing

ABSTRACT

The Aerial Refueling Scheduling Problem (ARSP) can be defined as determining the refueling completion times for fighter aircrafts (jobs) on multiple tankers (machines) to minimize the total weighted tardiness. ARSP can be modeled as a parallel machine scheduling with ready times and due date-to-deadline window to minimize total weighted tardiness. ARSP assumes that the jobs have different ready times and a due date-to-deadline window between refueling due date and a deadline to return without refueling. In this paper, we first formulate the ARSP as a mixed integer programming model. The objective function is a piece-wise tardiness cost that takes into account due date-to-deadline windows and job priorities. Since ARSP is NP-hard, two heuristics are proposed to obtain solutions in reasonable computation times, namely (1) modified ATC rule (MATC), (2) a simulated annealing method (SA). The proposed heuristic algorithms are tested in terms of solution quality and CPU time through computational experiments with data randomly generated to represent aerial refueling operations of an in-theater air operation. Solutions provided by both algorithms were compared to optimal solutions for problems with up to 12 jobs and to each other for larger problems with up to 60 jobs. The results show that, MATC is more likely to outperform SA especially when the problem size increases, although it has significantly worse performance than SA in terms of deviation from optimal solution for small size problems. Moreover CPU time performance of MATC is significantly better than SA in both cases.



## 1. Introduction

Aerial refueling (AR) is the process of transferring fuel from a tanker aircraft to a receiver aircraft during flight. AR is extensively used in large-scale military operations because of its advantages for an air force. In-theater aerial refueling is supported entirely through aerial refueling tracks (tracks 1 and 2 in Fig. 1), which are similar to gas stations floating in the sky. Empty alternative tracks 3–6 and wings 1–9 that are formed by various even numbers of aircrafts are also shown in Fig. 1. Tankers orbit in a track location with a constant speed and altitude waiting for receivers to arrive for refueling. The Aerial Refueling Scheduling Problem (ARSP) can be defined as determining the assignment of each fighter aircraft (job) to tanker (machine) and the refueling completion times for the aircrafts. ARSP assumes that aircraft wings stay together as a group, alternative track locations and assigned track stations (tracks 1 and 2) for the tankers are known, the number of tankers does not change during an operation (i.e., tankers replace each other without delay), and aircraft wings which move dynamically in the sky, can reach to available tankers in equal times.

Since the fighting force endurance in the air operation is much more important than the fuel costs of the air operation, ARSP was modeled as receiver-based. Thus, fuel source is assumed continuous to supply the receivers' demand without delay. ARSP can be modeled as an identical parallel machine scheduling problem with the tankers being machines and the aircraft fighters being jobs that have ready times (time they are available) and require certain amount of processing (refueling) time regardless to which tanker they are assigned. Minimizing the total weighted tardiness is a reasonable objective function to meet the aircrafts' refueling due dates and ultimately the mission's due date. Additionally, the aircrafts have a refueling deadline that they cannot miss; otherwise, they will have to go back to their base resulting in unscheduled jobs with a high cost. To effectively model both aspects of due dates and deadlines, a new piecewise tardiness cost function is defined to capture the cost structure over a time horizon that encompasses the due dates and deadlines. In this paper, a mixed integer linear programming (MILP) model will be developed to find optimal solutions for the problem. However, since the identical parallel machine scheduling is NP-hard even with only two machines

☆ This manuscript was processed by Area Editor Subhash C. Sarin.

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doi:10.1016/j.cie.2011.09.015

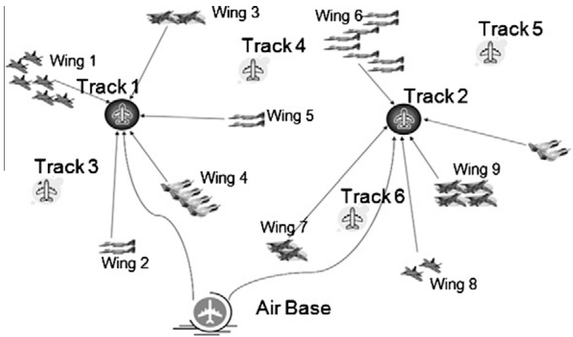


**Fig. 1.** In-theater aerial refueling.

(Blazewicz, Ecker, Pesch, Schmidt, & Weglarz, 2007; Garey & Johnson, 1979; Karp, 1972), ARSP is also NP-hard, which means that obtaining optimal solutions for large instances will be computationally difficult. Therefore, a composite dispatching rule, namely the Modified ATC (MATC) is proposed based on the commonly used Apparent Tardiness Cost (ATC) rule, which is often applied to total weighted tardiness problems. Dispatching (or priority) rules are very common heuristics for scheduling problems due to their easy implementation and low computational requirements. A Simulated Annealing (SA) metaheuristic is also developed for the problem at hand to compare the effectiveness of the proposed MATC rule for large instances.

The rest of this paper is organized as follows. In Section 2, the ARSP is defined and mapped to the abstract scheduling problem. Related research is summarized in Section 3. A MIP for the problem is developed in Section 4 and solution methods are introduced in Section 5. A computational study for small and large problem sizes is described in Section 6. Finally results are concluded in Section 7.

## 2. Problem definition

ARSP can be defined as scheduling $n$ jobs (wings of aircrafts) on $m$ identical parallel machines (tankers) where job $j$ arrives (becomes available) at ready time $r_j$ and should be complete by the due date $d_j$ and before deadline $D_j$. The problem elements are shown in Fig. 2. Job $j$ requires processing time $p_j$ which is the time required to approach the refueling area, anchoring to a tanker and fuel pumping starting at time $s_j$ and completing at time $C_j$. Ready time $r_j$ is the earliest time a receiver can start processing (i.e., receiver cannot be scheduled before $r_j$). The due date $d_j$ is a planned latest date of a receiver to complete refueling and the deadline $D_j$ is the latest date of a receiver to finish refueling after which it must return to base to avoid running too low on fuel level. Missing the due date is not preferred but allowed and a weighted tardiness cost will be incurred for jobs that miss their due date but not their deadline. Weight $w_j$ represents job priority for refueling. If a job misses $D_j$, it must return to base incurring a high penalty and will not be assigned to a machine. Due date-to-deadline window $(D_j - d_j)$ is the time window in which a weighted tardiness cost is incurred. There is also a scheduling window between ready times and deadlines in which all jobs have to be started and completed. The objective is to find a schedule that minimizes the total weighted tardiness (*TWT*) as a performance measure to maintain the quality of service with due dates.

ARSP assumes that the jobs are ready at different times $r_j$ and have a due date-to-deadline (d-to-D) window of different sizes. A piecewise tardiness cost function may be defined where if job $j$ is not completed on or before $d_j$, the tardiness cost $T_j = \max(C_j - d_j, 0)$ of job $j$ is incurred in due date-to-deadline window. The job is not scheduled and a high *unavailability cost, F* will be incurred, as shown in Fig. 3, if its completion time $C_j$ passes $D_j$.

## 3. Literature review

Aerial refueling is generally employed in two cases of military operations: inter-theater (e.g. overseas deployments) and in-theater (e.g. local conventional operations). This paper examines the in-theater ARSP which is far more complex and difficult to solve than inter-theater ARSP. The only existing research on scheduling in-theater aerial refueling is by Jin, Shima, and Schumacher (2006) who introduced the static autonomous refueling scheduling of multiple unmanned aerial vehicles (UAVs) on a single tanker to minimize the total time needed for refueling all UAVs in the sequence. A dynamic programming method was used to develop an efficient recursive algorithm to find the optimal initial sequence. On the other hand, the only major work on inter-theater aerial refueling is by Barnes, Wiley, Moore, and Ryer (2004) who studied the aerial fleet refueling problem (AFRP) and used a Group Theoretic Tabu Search (GTTS) approach as a solution method.

There is very little existing research addressing the scheduling identical parallel machines with ready times to minimize total weighted tardiness ($P_m|r_j|\Sigma w_jT_j$) problem as shown in Table 1. However, the related researches do not consider neither the d-to-D windows nor the piece-wise tardiness. The most closely related research is presented in Mönch, Balasubramanian, Fowler, and Pfund (2005), Reichelt, Mönch, Gottlieb, and Raidl (2006), Pfund, Fowler, Gadkari, and Chen (2008), Gharehgozli, Tavakkoli, and Zaerpour (2009), and Driessel and Mönch (2009) with the differentiators being the sequence dependent setup, batch machines, and precedence constraints.

Mönch et al. (2005) attempted to minimize total weighted tardiness on parallel batch machines with incompatible job families and unequal ready times ($P_m|r_j, batch, incomp.|\Sigma w_jT_j$). They proposed two different decomposition approaches. Dispatching and scheduling rules were used for the batching phase and the sequencing phase of the two approaches. Reichelt et al. (2006) were interested in minimizing total weighted tardiness and makespan at the same time ($P_m|r_j, batch, incomp.|\Sigma w_jT_j, C_{max}$). In order to determine a pareto efficient solution for the scheduling of jobs with incompatible families on parallel batch machines problem, they suggested a hybrid multi objective genetic algorithm. Pfund et al. (2008) addressed scheduling jobs with ready times on identical parallel machines with sequence dependent setups by

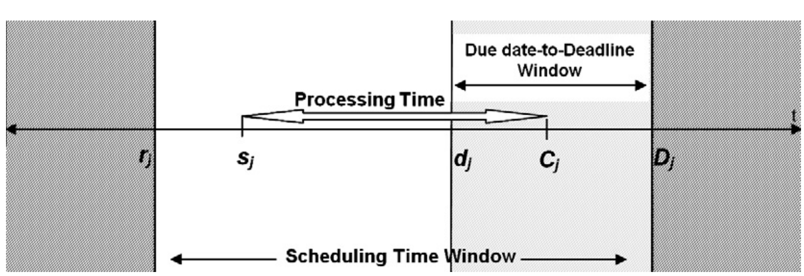


**Fig. 2.** Scheduling time horizon.