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Economic dynamics with financial fragility and mean-field interaction: A model

C. Di Guilmi[a,*], M. Gallegati[a], S. Landini[b]

[a] Department of Economics, Università Politecnica delle Marche, Piazzale Martelli 8, 60121 Ancona, Italy
[b] IRES Piemonte - Istituto Ricerche Economiche e Sociali del Piemonte, Via Nizza 18, 10125 Turin, Italy

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Abstract

Following Aoki's statistical mechanics methodology [Masanao Aoki, New Approaches to Macroeconomic Modeling, Cambridge University Press, 1996; Masanao Aoki, Modeling Aggregate Behaviour and Fluctuations in Economics, Cambridge University Press, 2002; Masanao Aoki, and Hiroshi Yoshikawa, Reconstructing Macroeconomics, Cambridge University Press, 2006], we provide some insights into the well-known works of [Bruce Greenwald, Joseph Stiglitz, Macroeconomic models with equity and credit rationing, in: R. Hubbard (Ed.), Information, Capital Markets and Investment, Chicago University Press, Chicago, 1990; Bruce Greenwald, Joseph Stiglitz, Financial markets imperfections and business cycles, Quarterly journal of Economics (1993)]. Specifically, we reach analytically a closed form solution of their models overcoming the aggregation problem. The key idea is to represent the economy as an evolving complex system, composed by heterogeneous interacting agents, that can be partitioned into a space of macroscopic states. This meso level of aggregation permits to adopt mean-field interaction modeling and master equation techniques.

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1. Introduction

Aim of the present work is to resettle the models introduced in Refs. [5,6] in a dynamic stochastic framework, as defined by Aoki [1–3]. The quantitative previsions of [6] (and of its developments [7]) replicate a number of stylized facts, strengthening the idea that the economy would be better represented as a complex dynamical system rather than a mere sum of identical and perfectly informed agents. They introduce indirect interaction of heterogeneous agents in the model of financial fragility presented in Ref. [5], originally conceived with a representative agent. This original model adopted a bankruptcy approach in emphasizing the causal relation among firms financial variables and aggregate fluctuations in the presence of asymmetric information. In financial markets, the presence of informational

* Corresponding author.
E-mail address: c.diguilmi@univpm.it (C. Di Guilmi).

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asymmetries among firms and investors gives rise to rationing of credit [8] and equity [9] for firms. According to bankruptcy approach, macroeconomic fluctuations arise for the rationing in equity market that obliges firms to finance investment with new debt. Accumulation of debt causes a growing risk of default. Since the optimal level of debt is different for different types of firms [10], also probabilities of bankruptcy come out to be different. Ref. [6] evidenced that a complex system structure appears to be particularly suitable to model a financial fragility approach, in which, given the existence of different financial structures, the distribution of agents matters. Any attempt to analytically resolve this kind of models must face the aggregation issue, since the problem of how to sum up heterogeneous and evolving agents cannot be dealt with the consuete tools of the economist. Masanao Aoki [1–3] managed to remove the representative agent hypothesis, introducing in economics the ground-breaking concept of mean-field interaction, that makes the analytical aggregation of heterogeneous agents feasible, replacing the unrealistic mechanic determinism of mainstream framework with a set stochastic tools borrowed from statistical mechanics. Mean-field interaction can be defined as the average interaction model that substitutes all the relations among agents that, otherwise, could not be analytically treated [11]. All agents are clusterized in a pre-defined set of states, basing on one particular feature that determines the characteristics and the evolution of the aggregate. Object of the analysis is not on the single agent, but the number or proportion of agents that occupy a certain state at a certain time. These levels are governed by a stochastic law, that also defines the functional of the probability distributions of aggregate variables and, if existing, their equilibrium distributions. The structure of the work is the following: firstly, we specify the hypothesis for the stochastic structure of the system (Section 2) and for the firms that compose it (Section 3); then (Section 4), we develop the probabilistic framework, setting the dynamical instruments needed for the aggregation.

2. Structure of the system and definition of states

We set up a model in continuous time for a system of heterogeneous and interacting agents, partitioned into groups or states. In this paragraph we state the hypothesis that are at the root of this stochastic dynamics, and in particular the definition and the structure of the states. Our system is articulated in two states. This partition permits to isolate the effect of bankruptcy costs on the aggregate dynamics. Moreover, the application of recently proposed methods for systems with higher order of states [12] would entail a too high price in terms of computational complication respect to the expected improvement in the realism of the model. Along time, a single firm can be in one of the two states, depending on its financial soundness, proxied by the equity ratio[1] (the ratio among the net worth and the total assets). Therefore there are two types of firms: the "good" firms, that have a high equity ratio, and the "bad" firms, that have a low equity ratio that exposes them to the risk of demise. System works in continuous time [13]: $t \in \mathbb{T} \subset \mathbb{R}_+$.

The economy is populated by a fixed number of firms $N = N(t) : \forall t \in \mathbb{T}$, each indexed by i for any given time.

The system's vector of states $\underline{\omega}$ is identified by the financial condition of the firm:

$$\underline{\omega}(t) = \{\omega_i(t) = HV\,(a_i(t)|\bar{a})\,\forall i \le N\} : HV(a_i(t)|\bar{a}) = \begin{cases} 1 \iff a_i(t) < \bar{a} \\ 0 \iff a_i(t) \ge \bar{a} \end{cases}$$

where $\bar{a}$ represents the threshold of equity ratio, that individuates firms that are in a critical financial situation, and, therefore, for which the probability of bankruptcy is bigger than 0 and where $a_i(t)$ is the equity ratio for firm i at time t.

In what follows we set x for firms which equity ratio is under the threshold and $a^0(t) = y$ for firms which equity ratio is over the threshold:

$$\omega = x \Leftrightarrow \omega_i(t) = 1 \vee \omega = y \Leftrightarrow \omega_i(t) = 0. \tag{1}$$

The cardinality of the jth state, to say the number of firms in state $j = 0, 1$ is given by

$$\left.\begin{array}{l} \text{card } x = \#\{\omega_i(t) = 1 \forall i \in I\} = N^1(t) \\ N^0(t) = N - N^1(t) \end{array}\right\} \Rightarrow \underline{N}(t) = \left(N^0(t), N^1(t)\right). \tag{2}$$

By assumption, the dynamics of the occupation number N^j follows a continuous time jump Markov process, defined over a state space $\Omega = (x, y)$ equipped with the following counting measure $N_{(.)}(.) : \Omega \times \mathbb{T} \to \mathbb{N}$, so that

[1] For a treatment of firms' financial variables as a predictor of bankruptcy see Ref. [14].