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Parallelization of population-based multi-objective meta-heuristics: An empirical study

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Abstract

In single-objective optimization it is possible to find a global optimum, while in the multi-objective case no optimal solution is clearly defined, but several that simultaneously optimize all the objectives. However, the majority of this kind of problems cannot be solved exactly as they have very large and highly complex search spaces. Recently, meta-heuristic approaches have become important tools for solving multi-objective problems encountered in industry as well as in the theoretical field. Most of these meta-heuristics use a population of solutions, and hence the runtime increases when the population size grows. An interesting way to overcome this problem is to apply parallel processing. This paper analyzes the performance of several parallel paradigms in the context of population-based multi-objective meta-heuristics. In particular, we evaluate four alternative parallelizations of the Pareto simulated annealing algorithm, in terms of quality of the solutions, and speedup.

Keywords: Parallel processing; Multi-objective optimization; Pareto simulated annealing; Graph partitioning

1. Introduction

Solving real optimization problems requires efficient algorithms. When the complexity of the problem to solve is high, such as with NP-complete problems [1], it is often useful to employ heuristic methods. Heuristics often allow us to tackle large-size problems instances by delivering satisfactory solutions in a reasonable

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runtime. Meta-heuristics are general-purpose heuristics that split into a number of categories including evolutionary algorithms (EA) and local-search methods (LS). While EAs allow a better exploration of the search space, LS strategies have the power to intensify the search in a particular region. However both, EA and LS, have in common that the quality of the solutions depends on parametric settings, like population size, number of iterations, etc. In many cases, the use of a single-processor computer implies very large runtimes, which can be unacceptable for some real applications. One way to overcome this weakness is the use of parallel processing.

Most real optimization problems entail simultaneous optimization of distinct and conflicting objectives. In recent years, the number of strategies proposed to solve complex multi-objective optimization problems has increased considerably. Population-based strategies are very frequently used in multi-objective meta-heuristics (MOMHs) [2–7]. In addition, the complexity of some of these methods [3,5,7] is even higher because they use a secondary population, which saves the promising solutions found in the search. These, and other parameters, such as the number of objectives to optimize, complicate even more the design of meta-heuristics in comparison with the single-objective case. In this context, the use of parallel and distributed strategies become a powerful tool to obtain good solutions in acceptable runtimes. Thus far, most of the parallel implementations in multi-objective optimization are related to EA-based MOMHs, while LS-based MOMHs have been studied to a lesser extent. This paper analyzes the behavior of four parallel paradigms in Pareto simulated annealing, a well known population-based MOMH.

The remainder of the paper is organized as follows. Section 2 describes the multi-objective optimization concept, and reviews the main simulated annealing-based MOMHs proposed to date. Section 3 gives an overview of parallel and distributed strategies, and details how to extend them to the analysis of population-based MOMHs. Section 4 presents the results of the parallel models proposed in Section 3, in a multi-objective formulation of the graph-partitioning problem. Finally, Section 5 provides the conclusions of this empirical analysis.

2. Multi-objective optimization: aims and methods

2.1. General concepts in multi-objective optimization

Although single-objective optimization (SOO) methods model many real problems, there exist a large number of applications where they are inappropriate, because it is nearly impossible to obtain a single solution which simultaneously optimizes all the objectives. In the last years, authors have proposed several multi-objective optimization (MOO) strategies to overcome this situation. In the following, we present some general concepts of MOO.

Definition 1. *Multi-objective optimization* (*MOO*) is the process of searching one or more decision variables that simultaneously satisfy all constraints, and optimize an objective function vector that maps the decision variables to two or more objectives.

Definition 2. *Decision vector or solution* $(s) = (s_1, s_2, \ldots, s_n)$ represents accurate numerical qualities for an optimization problem. The set of all decision vectors constitutes the *decision space*.

Definition 3. *Feasible set* (F) is the set of decision vectors that simultaneously satisfies all the constraints.

Definition 4. *Objective function vector* (f) maps the decision vectors from the decision space into a *K-dimensional* objective space $Z \in \Re^K$, $z = f(s)$, where $f(s) = \{f_1(s), f_2(s), \ldots, f_K(s)\}$, $z \in Z$, $s \in F$.

Definition 5. *Multi-objective optimization—mathematical form*

$$\begin{aligned}&\text{minimize/maximize}(f_k(s)) \quad \forall k \in [1, K]\\&\text{subject to } s \in F.\end{aligned}$$