Information Sciences 220 (2013) 507–521

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Multi-objective portfolio selection model with fuzzy random returns and a compromise approach-based genetic algorithm

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ARTICLE INFO

Article history:
Received 28 April 2010
Received in revised form 9 July 2012
Accepted 15 July 2012
Available online 24 July 2012

Keywords:
Portfolio selection
Multi-objective programming
Fuzzy random variable
Compromise solution
Genetic algorithms

ABSTRACT

This paper addresses the multi-objective portfolio selection model with fuzzy random returns for investors by studying three criteria: return, risk and liquidity. In addition, securities historical data, experts' opinions and judgements and investors' different attitudes are considered in the portfolio selection process, such that the investor's individual preference is reflected by an optimistic–pessimistic parameter λ. To avoid the difficulty of evaluating a large set of efficient solutions and to ensure the selection of the best solution, a compromise approach-based genetic algorithm has been designed to solve the proposed model. In addition, a numerical example is presented to illustrate the proposed algorithm.

1. Introduction

Modern portfolio selection theory originated from the pioneering research work of Markowitz's mean–variance model [35]. Based on the mean–variance model, many scholars proposed model extensions by assuming the securities' rates of return were random variables and thus only used historical data to describe the securities future rates of return. However, in addition to random uncertainty, there are many non-probability factors in the securities market that cannot be resolved using probability theory. With the introduction of fuzzy set theory [50,51], some authors have developed fuzzy portfolio selection models (cf. [6,14,17,18,25,28,47,48,52,2] and the references therein). These authors recognized the existence of fuzziness in the securities market but ignored other categories of uncertainty because only fuzzy uncertainty is reflected in the research.

In a complicated financial market, some variables can exhibit random uncertainty properties and others can exhibit fuzzy uncertainty properties. Because random uncertainty and fuzzy uncertainty are often combined in a real-world setting, the portfolio selection process must simultaneously consider twofold uncertainty. Katagiri and Ishii [19] first assumed securities' rates of returns were fuzzy random variables and proposed a portfolio selection model based on possibility theory and a chance-constrained model in stochastic programming. Smimou et al. [41] presented a method for the derivation of the attainable efficient frontier in the presence of fuzzy information in data. Li and Xu [26] proposed the λ-mean variance portfolio selection model based on fuzzy random theory. Yoshida [49] discussed a value-at-risk portfolio model of randomness and fuzziness to derive its analytical solution. Lacagnina and Pecorella [24] developed a multistage stochastic soft constraints fuzzy program with the goal of capturing both uncertainty and imprecision as well as to re-solving a portfolio management issue.

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0020-0255/$ - see front matter
http://dx.doi.org/10.1016/j.ins.2012.07.005

Expected return and risk are two fundamental factors in portfolio selection. However, explicit return and risk cannot capture all relevant information for an investment decision. Therefore, criteria for portfolio selection problems, in addition to the standard expected return and variance, have become more popular in recent years [14]. Steuer et al. [43] discussed portfolio selection for investors using a multi-objective stochastic programming problem. Parra et al. [36] proposed a portfolio selection model with the three criteria (return, risk and liquidity) and resolved the model using a fuzzy goal programming approach. Fang et al. [11] presented a portfolio rebalancing model with three criteria (return, risk and liquidity) based on fuzzy decision theory. Gupta et al. [14] studied a semi-absolute deviation portfolio selection model, intended for investors' that incorporates five criteria (short-term return, long-term return, dividend, risk and liquidity). However, realistic constraints are not considered in the above-cited works. Because of the existence of realistic constraints, it is difficult to resolve constrained multi-objective portfolio selection models using traditional multi-objective programming algorithms. Some authors use evolutionary algorithms to resolve constrained multi-objective portfolio optimization models. Ehrgott et al. [10] used a genetic algorithm to optimize a mixed-integer (due to the constraints used) multi-objective portfolio optimization problem with objectives aggregated through user-specified utility functions. Subbu et al. [46] presented a hybrid evolutionary algorithm that integrated genetic algorithms with linear programming for a portfolio design issume with multiple measures for risk and return.

In this paper, we propose a constrained multi-objective portfolio selection model with fuzzy random returns for investors. This model includes three criteria (return, risk and liquidity) and a compromise approach-based genetic algorithm designed to obtain a compromised portfolio strategy. The model has the ability to introduce expert opinion and judgment (fuzzy information) into the portfolio selection process and to obtain a satisfactory personal portfolio selection in accordance with the attitudes of the different investors'. The rest of the paper is organized as follows. In Section 2, definitions for fuzzy random variable, fuzzy expectation and variance of fuzzy random variables are briefly introduced. In Section 3, we use the λ-average value of the fuzzy expectation of portfolio to quantify the return, the variance to quantify risk, and the crisp possibilistic mean value of the turnover rate portfolio to quantify portfolio liquidity. Then, a constrained multi-objective portfolio selection model with fuzzy random returns is proposed. In Section 4, to avoid the difficulty of evaluation and the selection of the best solution from the efficient frontier or its discretized representation, a compromise approach-based genetic algorithm has been designed to resolve the proposed model and to obtain a compromised portfolio strategy. An example is given in Section 5 to illustrate the proposed model and algorithm, and concluding remarks are given in Section 6.

2. Preliminaries

A fuzzy number $\widetilde{X}$ is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\widetilde{X}} : \mathbb{R} \to [0,1]$ satisfies the following conditions:

(i) $\widetilde{X}$ is normal, i.e., there exists an $x \in \mathbb{R}$ such that $\mu_{\widetilde{X}}(x) = 1$;
(ii) $\mu_{\widetilde{X}}$ is quasi-concave, i.e., $\mu_{\widetilde{X}}(\lambda x + (1-\lambda)y) \leqslant \min\{\mu_{\widetilde{X}}(x), \mu_{\widetilde{X}}(y)\}$, for all $\lambda \in [0,1]$;
(iii) $\mu_{\widetilde{X}}$ is upper semi-continuous, i.e., $\{x \in \mathbb{R} | \mu_{\widetilde{X}}(x) \leqslant \alpha\}$ is a closed set, for all $\alpha \in [0,1]$; and
(iv) the closure of the set $\{x \in \mathbb{R} | \mu_{\widetilde{X}}(x) > 0\}$ is a compact set.

An α-level set of $\widetilde{X}$ is defined by $\widetilde{X}_\alpha = \{x \in \mathbb{R} | \mu_{\widetilde{X}}(x) \geqslant \alpha\}$ if $\alpha > 0$ and $\widetilde{X}_\alpha = \mathrm{cl}\{x \in \mathbb{R} | \mu_{\widetilde{X}}(x) > 0\}$ (the closure of the support of $\widetilde{X}$) if $\alpha = 0$. It is well known that if $\widetilde{X}$ is a fuzzy number, then $\widetilde{X}_\alpha = [\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+]$ is a compact subset of $\mathbb{R}$ for all $\alpha \in [0,1]$.

The concept of fuzzy random variable, which was first introduced by Kwakernaak [23], applies to a situation when randomness and fuzziness appear simultaneously.

Definition 1. ([37]) Let $(\Omega, \mathscr{A}, P)$ be a probability space, where $\mathscr{A}$ is a σ-field of Ω and P is a non-atomic probability measure. A mapping $\widetilde{\widetilde{X}} : \Omega \to \mathscr{F}_c(\mathbb{R})$ is said to be a fuzzy random variable if the set-valued function $\widetilde{\widetilde{X}}_\alpha : \Omega \to \mathscr{K}_c(\mathbb{R})$ such that $\widetilde{\widetilde{X}}_\alpha(w) = (\widetilde{\widetilde{X}}(w))_\alpha = \{x \in \mathbb{R} | \mu_{\widetilde{\widetilde{X}}(w)}(x) \geqslant \alpha\}$ for all $w \in \Omega$ is $\mathscr{A}$-measurable for all $\alpha \in [0,1]$, where $\mathscr{F}_c(\mathbb{R})$ denotes the set of all fuzzy numbers, and $\mathscr{K}_c(\mathbb{R})$ denotes the class of all non-empty bounded closed intervals.

As shown in [33], if $\widetilde{\widetilde{X}}$ is a fuzzy random variable, the left endpoint $(\widetilde{\widetilde{X}}(w))_\alpha^-$ and the right endpoint $(\widetilde{\widetilde{X}}(w))_\alpha^+$ of the α-level sets of $\widetilde{\widetilde{X}}(w)$ are real-valued random variables for all $\alpha \in (0,1]$.

Example 1. Let L, R:$[0,1] \to [0,1]$ be continuous and strictly decreasing functions with $R(0) = L(0) = 1$ and $R(1) = L(1) = 0$. A fuzzy random variable $\widetilde{\widetilde{X}}$ characterized by the membership function

$$\mu_{\widetilde{\widetilde{X}}(w)}(x) = \begin{cases} L\left(\frac{a(w)-x}{\alpha}\right), & \text{if } a(w) - \alpha \leqslant x \leqslant a(w), \\ R\left(\frac{x-a(w)}{\beta}\right), & \text{if } a(w) < x \leqslant a(w) + \beta, \end{cases} \quad \forall w \in \Omega,$$

is called an L–R type fuzzy random variable, where random variable $a(w)$ is the center value and positive real numbers α and β are the left width and right width of the fuzzy number $\widetilde{\widetilde{X}}(w), w \in \Omega$, respectively. For simplicity, $\widetilde{\widetilde{X}}$ is denoted by $\widetilde{\widetilde{X}}(w) = (a(w), \alpha, \beta)_{LR},\ w \in \Omega$ (see Fig. 1).