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Fast computation of accurate Gaussian–Hermite moments for image processing applications

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ABSTRACT

Gaussian–Hermite moments are orthogonal moments widely used in image processing and computer vision applications. Similar to the other families of orthogonal moments, highly computational demands represent the main challenging. In this work, an efficient method is proposed for fast computation of highly accurate Gaussian–Hermite moments for gray-level images. The proposed method achieves the accuracy through the integration of Gaussian–Hermite polynomials over the image pixels. To achieve the efficiency, the symmetry property of Gaussian–Hermite polynomials is employed where the computational complexity is reduced by 75%. Fast computational methodology is employed to significantly accelerate the computational process where the 2D Gaussian–Hermite moments are treated in a separated form. Numerical experiments are performed where the results are compared with the conventional method. The comparison of the obtained results clearly ensures the efficiency of the proposed method.

1. Introduction

Moments and functions of moments have been widely used in different applications of pattern recognition, image processing and computer vision [1–6]. Geometric moments and their translation, scaling and rotation invariants were firstly introduced and implemented by Hu [7]. In the early 80's of the last century, Teague [8] introduced the concept of orthogonal moments, where continuous orthogonal polynomials are used to generate moments for image analysis. Legendre, Zernike and pseudo Zernike are examples of continuous orthogonal moments. Teh and Chin [9] showed that, orthogonal moments could be used to represent an image with the minimum amount of information redundancy.

Gaussian–Hermite moments represent another kind of continuous orthogonal moments. These moments were firstly introduced by Shen [10]. Shen et al. [11] compared the performance of Gaussian–Hermite moments as orthogonal moments and the geometric moments. Later, Wu and Shen [12] discussed the properties of the orthogonal Gaussian–Hermite moments and their applications. Gaussian–Hermite moments are used in detection of the moving objects [13–15], fingerprint segmentation and classifications [16–18], medical image segmentation [19], stereo matching [20], image denoising [21], iris recognition [22] and license plate character recognition [23]. Recently, Yang and his co-authors [24] derived the rotation and translation invariants of Gaussian–Hermite moments. It is clear that, the computational process of rotation and translation Gaussian–Hermite moment invariants is dependent on the computation of the original Gaussian–Hermite moments.

The conventional computation of continuous orthogonal moments includes numerical approximation which results in by replacing integration by a truncated finite summation. Liao and Pawlak [25] attempted to overcome this problem by using a modified approximation method. Recently, exact computation of moments by integrating their polynomial functions over image pixels is an elegant approach proposed by Hosny for efficient and accurate computation of geometric moments [26], Legendre moments [27], radial moments [28], and Zernike moments [29,30].

This paper proposes a fast method for accurate computation of orthogonal Gaussian–Hermite moments for binary and gray-level images. The 2D Gaussian–Hermite moments are computed by applying the approach of mathematical integration of Gaussian–Hermite functions over digital image pixels where the approximation errors are avoided. The symmetry property of Gaussian–Hermite functions is employed to achieve a significant reduction in the computational process. The conducted numerical experiments clearly show the efficiency of the proposed method.

The rest of the paper is organized as follows: In Section 2, a concise presentation of orthogonal Gaussian–Hermite moments and their approximation computation is given. In Section 3, a detailed description of the proposed method for computation of accurate Gaussian–Hermite moments is presented. Section 4 is devoted to numerical experiments and results. Conclusion is presented in Section 5.

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2. Gaussian–Hermite moments

Hermite polynomials are orthogonal polynomials defined over the domain $(-\infty, \infty)$ where the Hermite polynomial of degree p is defined as follows [31]:

$$H_p(x) = (-1)^p e^{x^2} \frac{d^p}{dx^p}\left(e^{-x^2}\right) \tag{1}$$

Such polynomial could be defined in an explicit expansion as follows:

$$H_p(x) = p! \sum_{m=0}^{\lfloor \frac{p}{2} \rfloor} (-1)^m \frac{1}{m!(p-2m)!} (2x)^{p-2m} \tag{2}$$

where the operator $\lfloor p/2 \rfloor$ is equal to $(p-1)/2$ if p is odd otherwise equal to $p/2$. The recurrence relation of Hermite polynomials is:

$$H_{p+1}(x) = 2xH_p(x) - 2pH_{p-1}(x) \tag{3}$$

where $p \geqslant 1$ and the first two polynomials are $H_0(x) = 1$ and $H_1(x) = 2x$. The orthogonality relation of Hermite polynomials with respect to the weight function, e^{-x^2}, is defined as follows:

$$\int_{-\infty}^{\infty} e^{-x^2} H_p(x) H_q(x)\, dx = 2^p p! \sqrt{\pi}\, \delta_{pq} \tag{4}$$

Based on Eq. (4), the normalized Hermite polynomials are defined using original Hermite polynomials as follows:

$$\hat{H}_p(x) = \frac{1}{\sqrt{2^p p! \sqrt{\pi}}} e^{\left(-\frac{x^2}{2}\right)} H_p(x) \tag{5}$$

The normalized Hermite polynomials satisfy the following orthogonality property:

$$\int_{-\infty}^{\infty} \hat{H}_p(x) \hat{H}_q(x)\, dx = \delta_{pq} \tag{6}$$

Replacing x by x/σ, Gaussian–Hermite functions are defined using the normalized Hermite function as follows:

$$\hat{H}_p(x/\sigma) = \frac{1}{\sqrt{2^p p! \sigma \sqrt{\pi}}} e^{\left(-\frac{x^2}{2\sigma^2}\right)} H_p(x/\sigma) \tag{7}$$

The parameter σ is the standard deviation. Gaussian–Hermite moments of order $(p+q)$ for the image intensity function, $f(x, y)$, is defined as:

$$M_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \hat{H}_p(x/\sigma) \hat{H}_q(y/\sigma)\, dx\, dy \tag{8}$$

For a digital image of size $M \times N$, the approximated Gaussian–Hermite moments are computed by using the following formula [32]:

$$\tilde{M}_{pq} = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(x_i, y_j) \hat{H}_p(x_i/\sigma) \hat{H}_q(y_j/\sigma) \Delta x \Delta y \tag{9}$$

In this formula, the double integration in Eq. (8) is replaced by double summations which results in numerical error. Based on the principles of mathematical analysis, summations are equivalent to integrals as the number of sampling points tends to infinity which is impossible in the limited computing environment. The numerical error increased as the number of sampling points decreased. Also, this error increased as the order of moments increased. Therefore, numerical instabilities could be encountered when the moment order reaches a cretin value. The optimum way to overcome this problem is the accurate evaluation of the double integration in Eq. (8).

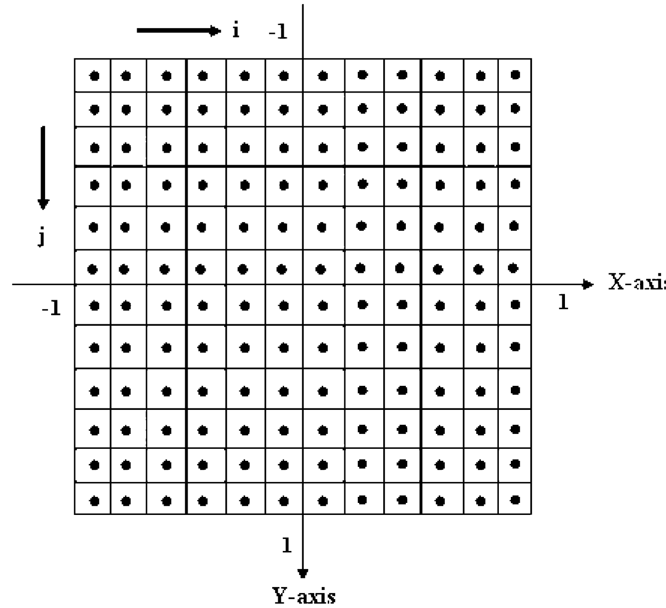

Fig. 1. Input image defined in the square $[-1, 1] \times [-1, 1]$.

3. The proposed method

Direct computation of Gaussian–Hermite moments by using Eq. (9) is impractical where two major challenges were raised. Inaccurate computed moments represent the first challenge while the highly computational costs represent the second challenge. The proposed method aims to overcome these two problems by presenting a fast and exact-like computation of Gaussian–Hermite moments.

To achieve these goals, the input digital image of size $M \times N$ is defined as an array of pixels. Centers of these pixels are the points (x_i, y_j), where the image intensity function is defined only for this discrete set of points $(x_i, y_j) \in [-1, 1] \times [-1, 1]$ as displayed in Fig. 1. $\Delta x_i = x_{i+1} - x_i$, $\Delta y_j = y_{j+1} - y_j$ are sampling intervals in the x- and y-directions respectively. In the literature of digital image processing, the intervals Δx_i and Δy_j are fixed at constant values $\Delta x_i = 2/M$, and $\Delta y_j = 2/N$ respectively. The points (x_i, y_j) are defined where $x_i = -1 + (i - 0.5)\Delta x$; $y_j = -1 + (j - 0.5)\Delta y$; $i = 1, 2, 3, \ldots, M$, and $j = 1, 2, 3, \ldots, N$.

3.1. Accurate Gaussian–Hermite moments

By substituting Eq. (2) in (7), the Gaussian–Hermite functions could be rewritten as follows:

$$\hat{H}_p(x/\sigma) = \frac{1}{\sqrt{2^p p! \sigma \sqrt{\pi}}} e^{\left(-\frac{x^2}{2\sigma^2}\right)} \sum_{m=0}^{\lfloor \frac{p}{2} \rfloor} (-1)^m \frac{p!}{m!(p-2m)!} (2x)^{p-2m} \tag{10}$$

Rearrange the mathematical terms in the right-hand side, Eq. (10) could be rewritten as follows:

$$\hat{H}_p(x/\sigma) = C_p(\sigma) \sum_{n=0}^{\lfloor \frac{p}{2} \rfloor} B_{p,m} \left(\frac{x}{\sigma}\right)^{p-2m} e^{\left(-\frac{x^2}{2\sigma^2}\right)} \tag{11}$$