On the choice of the parameters for anisotropic diffusion in image processing

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Abstract
Anisotropic diffusion filtering is highly dependent on some crucial parameters, such as the conductance function, the gradient threshold parameter and the stopping time of the iterative process. The various alternative options at each stage of the algorithm are examined and evaluated and the best choice is selected. An automatic stopping criterion is proposed, that takes into consideration the quality of the preserved edges as opposed to just the level of smoothing achieved. The proposed scheme is evaluated with the help of real and simulated images, and compared with other state of the art schemes using objective criteria.

1. Introduction

Based on the importance of the scale-space representation of images, which was introduced by Witkin [1], Perona and Malik suggested a new definition of scale-space through Anisotropic Diffusion (AD), a non-linear partial differential equation-based diffusion process [2]. Overcoming the undesirable effects of linear smoothing filtering, such as blurring or dislocating the semantically meaningful edges of the image, AD has become a very useful tool in image smoothing, edge detection, image segmentation and image enhancement. AD filtering can successfully smooth noise while respecting the region boundaries and small structures within the image, as long as some of its crucial parameters are determined or estimated correctly. The conductance function, the gradient threshold parameter and the stopping time of the iterative process form a set of parameters which define the behavior and the extent of the diffusion. Overestimating one of the parameters may lead to an oversmoothed blurry result, while underestimating it may leave the noise in the image unfiltered. Therefore, it is crucial that all parameters are determined in an optimal and automatic way in every step of the iterative process, by evaluating both the denoising needs and the quality of the edges of a given image.

Over the last years, a great amount of work has been done with respect to both the continuous and discrete form of AD filtering as stated by Perona and Malik [2]. As for the behavior of the continuous form of AD, there has been a considerable amount of research proving the ill-posedness of the diffusion equation and developing new well-posed equations or regularizing methods [3–11]. In [12,13] a semi-implicit scheme was presented, while in [14] a different discrete implementation was proposed in order to obtain better isotropy. A multigrid approach leading to a well-posed, steady-state solution was proposed in [11]. In [15] a time-dependent numerical scheme was proposed. In [16] a modified method that considers also the variance of the brightness levels in a local neighborhood around each pixel was presented. However, the problem of the automatic estimation of the crucial parameters was not addressed. A modified diffusion scheme, suitable for images with low-contrast and uneven illumination, was described in [17]. In [18,19] the attention was drawn mostly to the discrete implementation of the scheme and the experimental results of the new conductance functions that were proposed. The automatic estimation of the method’s parameters was also studied in these works. Since several conductance functions can be used, differentiating considerably the filtering results as shown in [18], it is necessary to define the appropriate one and scale it in a way that the edges remain the sharpest possible. The importance of the scaling of the conductance function is emphasized in the current work, leading to a comparison and selection of the most edge-preserving function.

In [20] the necessity of the gradient threshold parameter to be a decreasing function of time was first shown. In this way, the parameter adapts itself to the denoising needs of the filtered image after every iteration, preserving all the edges above a decreasing threshold. Various methods estimating this parameter were proposed using statistical characteristics of the image [2,18,19] and morphological operators [21]. These methods are compared in the current work, along with a proposed statistical...
method which estimates two gradient threshold parameters and yields robust filtering results.

Since AD is an iterative process, the problem of choosing the optimal time to stop the iterations and prevent an oversmoothed result is crucial. Adding a fidelity term that keeps the resultant image close to the original image has been proposed in [8,22,23] but the noise in that case had not been sufficiently removed. In [24], a stabilized, insensitive to the number of iterations process was introduced. A multigrid algorithm was presented and evaluated in [11], introducing a Brent-NCP (Normalised Cumulative Periodogram) automatic stopping parameter method. A frequency approach of the problem was presented in [25]. Some criteria estimating a stopping parameter have been introduced in [26–29] based solely on the extend of the noise smoothing of the filter in every iteration. Spatially varying stopping methods that increase significantly the computational cost were presented in [30,31]. The quality of the preserved edges was not considered in any of the above methods. As it is shown in the present work, the evaluation of the image’s edges is strongly related to the stopping time estimation problem. A novel automatic stopping criterion based on this approach is described and evaluated.

Anisotropic diffusion has been widely used in biomedical imaging [14,32–35]. Plenty of applications where nonlinear diffusion filtering has also been used can be found in [9].

The goal of the current work is to investigate fully the role of the parameters on the quality of the results of the AD discrete scheme and propose novel methodology for their automatic adaptation, as well as a novel termination criterion for the whole iterative process, so that the final denoised result is optimal.

This paper is organized as follows: in Section 2 we present the critical steps of the method and perform a comparative evaluation of the various proposed options. The right choice and scaling of the conductance function and the methods for estimating the gradient threshold parameter are considered in order to come up with the optimal automatic discrete scheme. In Section 3 we present a novel stopping criterion based on the quality of the image’s edges and in Section 4 we evaluate the scheme using a set of natural images. The discussion and concluding remarks are presented in Section 5.

2. Anisotropic diffusion

2.1. Overview of the process

The basic equation of anisotropic diffusion equation as presented in [2] is

\[
\frac{\partial I(x,y,t)}{\partial t} = \text{div}(g(\nabla I(x,y,t)) \nabla I(x,y,t))
\]

where \( t \) is the time parameter, \( I(x,y,0) \) is the original image, \( \nabla I(x,y,t) \) is the gradient of the image at time \( t \) and \( g(\cdot) \) is the so-called conductance function. This function is chosen to satisfy \( \lim_{x \to 0} g(x) = 1 \), so that the diffusion is maximal within uniform regions, and \( \lim_{x \to \infty} g(x) = 0 \), so that the diffusion is stopped across edges. Two such functions proposed by Perona and Malik were

\[
g_1(x) = \exp \left[ -\left( \frac{x}{K} \right)^2 \right]
\]

and

\[
g_2(x) = \frac{1}{1 + (\frac{x}{K})^2}
\]

where \( K \) is the gradient magnitude threshold parameter that controls the rate of the diffusion and serves as a soft threshold between the image gradients that are attributed to noise and those attributed to edges. Black et al. in [18], through an interpretation of AD in terms of robust statistics, defined a different conductance function, called Tukey’s biweight function

\[
g_3(x) = \begin{cases} 
\frac{1}{2} \left[ 1 - \left( \frac{x}{S} \right)^2 \right]^2, & x \leq S \\
0, & \text{otherwise}
\end{cases}
\]

where \( S = K \sqrt{2} \).

The flow function \( \phi \) defined as

\[
\phi(x) = g(x) x
\]

represents the sum of the brightness flow that is generated. The maximum flow is generated at locations where \( |\nabla I| = K \).

Perona and Malik discretized their anisotropic diffusion equation to

\[
I_{s+1}(s) = I_s(s) + \sum_{p \in \eta_4} \frac{\lambda}{|\eta_4|} \sum_{p \in \eta_4} g_1(\nabla I(p)) |\nabla I(p)|
\]

where \( I \) is a discretely sampled image, \( s \) denotes the pixel position in the discrete 2-D grid, \( t \) denotes the iteration step, \( g \) is the conductance function and \( K \) is the gradient threshold parameter. Constant \( \lambda \in (0,1] \) determines the rate of diffusion and \( \eta_4 \) represents the spatial 4-pixel neighborhood of pixel \( s \), \( \eta_4 = \{N, S, E, W\} \), where \( N, S, E \) and \( W \) are the North, South, East and West neighbors of pixel \( s \), respectively. Consequently, \(|\eta_4| = 4 \) (except for the image borders). The symbol \( \nabla \) which in the continuous form is used for the gradient operator, now represents a scalar defined as the difference between neighbouring pixels in each direction:

\[
\nabla I(p) - I_s(s), \quad p \in \eta_s = \{N,S,E,W\}.
\]

As mentioned by Perona and Malik, this scheme is not the exact discretization of the continuous equation, with more numerically consistent methods having been proposed in the literature. However, it is favoured due to its low computational complexity, preserving most of the properties of the continuous form [2].

2.2. Choosing the conductance function

According to Perona and Malik [2], the \( g_1 \) conductance function favours high-contrast edges over low-contrast ones, while the \( g_2 \) function favours wide regions over smaller ones. The \( g_2 \) function, according to Black et al. [18], yields sharper edges improving considerably the experimental results of the filtering, since the diffusion process converges faster. We will focus on the scaling and comparison done by Black et al. in order to examine the different experimental behavior of the conductance functions.

In order to be able to compare their efficiency, Black et al. scaled the conductance functions \( g_1 \), \( g_2 \) and \( g_3 \), so that their respective flow functions, given by (5), reach the same maximum value (producing the same amount of brightness flow) at the same point \( x = K \) (Fig. 1a). This leads us to the following scaled conductance functions:

\[
g_1(x) = \exp \left[ -\left( \frac{x}{K \sqrt{2}} \right)^2 \right]
\]

\[
g_2(x) = \frac{1}{1 + (\frac{x}{K})^2}
\]
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